

Physics near the conformal boundary in SU(3) gauge theory

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- at BNL RIKEN/HET joint seminar -

Jan. 9, 2014



“Higgs boson”

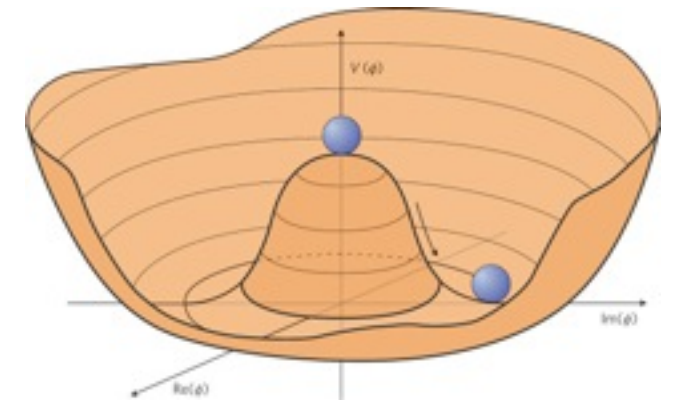
- Higgs boson found at LHC
- $m_H = 125 \text{ GeV}$
- so far consistent with Standard Model Higgs ($J^{PC}=0^{++}$) fundamental scalar
- but it could be different
- one of the possibilities:
 - composite Higgs
 - SM Higgs is the low energy effective description of that, cf: ChPT \Leftrightarrow QCD

Role of SM Higgs

- It's about the origin of mass...
- (99% of the mass of visible universe is made by QCD dynamics)
- masses of fundamental particles: quarks, leptons, weak bosons
 - by EW gauge symmetry breaking through Higgs

Higgs mechanism (cf. Farhi & Susskind)

- Higgs potential : $V = \mu^2 |\phi|^2 + \lambda |\phi|^4$ with $\mu^2 < 0$: “wine bottle”
 - rotating: $m=0$ mode
 - radial: $m \neq 0$: Higgs particle
- weak doublet: 4 fields: 1 massive Σ , 3 massless
- massless: Π^\pm, Π^0 : Nambu-Goldstone boson (rotational symm. br.)
- have coupling to weak current: $\langle 0 | J_\mu^\pm | \Pi^\pm \rangle = F p_\mu$; $F = \langle 0 | \phi | 0 \rangle = 246 \text{ GeV}$
- make a massless pole in the vacuum polarization
- cancels massless pole of original W^\pm propagator \rightarrow massive gauge boson



$$\langle 0 | J_\mu^\pm | \Pi^\pm \rangle = F p_\mu$$

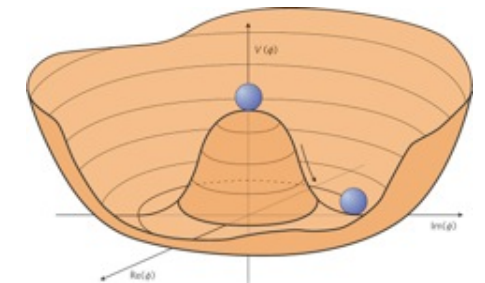
- Isn't it familiar ? : $\langle 0 | J_\mu^\pm | \Pi^\pm \rangle = F p_\mu$ with massless boson Π^\pm
- pion decay: $\langle 0 | A_\mu^\pm | \pi^\pm \rangle = f p_\mu$
 - π^\pm π^0 Nambu-Goldstone boson made of u, d quarks due to
 - $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$: spontaneous chiral symmetry breaking
 - in the real world: pseudo NG boson
 - $f=93 \text{ MeV} \Leftrightarrow F=246 \text{ GeV}$
- axial current A_μ^\pm is a part of weak current J_μ^\pm : (V-A)
- Even if there is no Higgs, weak boson gets massive due to chiral br. in QCD

Technicolor (TC)

- $\langle 0 | J_\mu^\pm | \Pi^\pm \rangle = F p_\mu$
 - realize this with a new set of
 - massless quarks (techni-quarks)
 - which have coupling to weak bosons,
 - and interact with techni-gluons
 - which breaks the chiral symmetry in the techni-sector,
 - produces techni-pions which have decay constant
- ➔ $F = 246 / \sqrt{N}$ GeV: scale up version of QCD (N: # weak doublet from new techni-sector)

Technicolor \Leftrightarrow SM Higgs

- success of technicolor
 - explaining the origin of EW symmetry breaking
 - dynamics of gauge theory $\Leftrightarrow \mu^2 < 0$
 - evading the gauge hierarchy problem: naturalness problem
 - due to logarithmic UV divergence \Leftrightarrow power divergence
- fermion masses ?
 - ETC effective 4 Fermi interaction \Leftrightarrow fermion-Higgs Yukawa coupling
 - produced by introducing interaction: techni-quarks and SM fermions

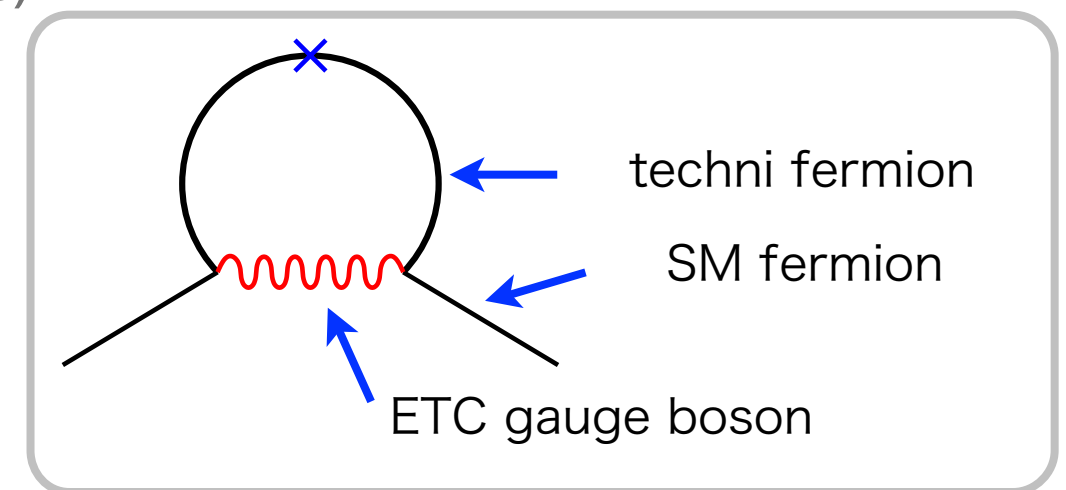


Extended Technicolor (ETC)

- fermion masses \rightarrow extended technicolor (ETC)
- New strong interaction of $SU(N_{ETC})$: $N_{ETC} > N_{TC}$, $T_{ETC} = (T, f)$: $T \in TC$, $f \in SM$
- SSB: $SU(N_{ETC}) \rightarrow SU(N_{TC}) \times SM$ @ $\Lambda_{ETC} (\gg \Lambda_{TC})$

- $$\frac{1}{\Lambda_{ETC}^2} \bar{T} T \bar{f} f \rightarrow m_f = \frac{\langle \bar{T} T \rangle_{ETC}}{\Lambda_{ETC}^2}$$

- $$\frac{1}{\Lambda_{ETC}^2} \bar{f} f \bar{f} f \quad \text{FCNC}$$



- FCNC should be small \Leftrightarrow top or bottom quark mass should be produced

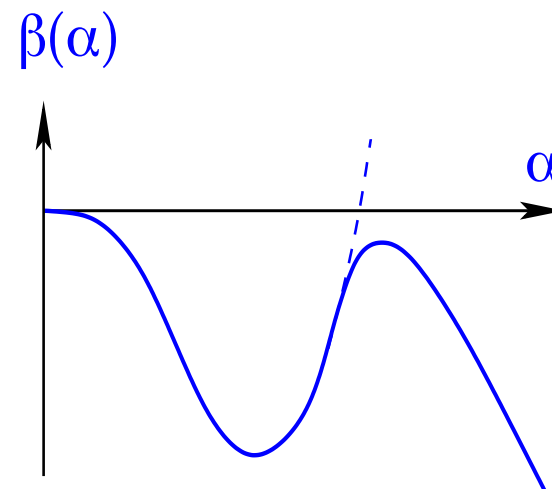
➡ walking TC

Walking Technicolor

- key: to realize suppressed FCNC and appropriate size of fermion masses

[Holdom, Yamawaki-Bando-Matsumoto]

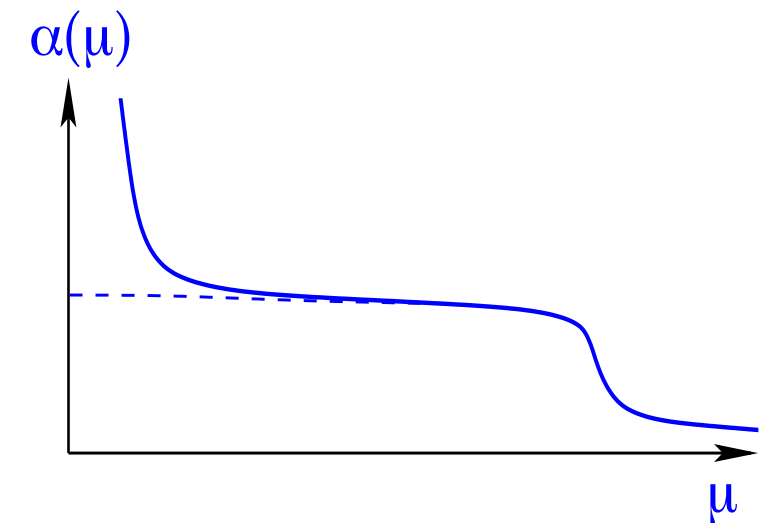
- renormalized gauge coupling
 - to **run very slowly** (walking)



- eventually grows at low energies \rightarrow to produce techni-pions

- mass anomalous dimension

- **large:** $\gamma_m \sim 1$



Walking Technicolor

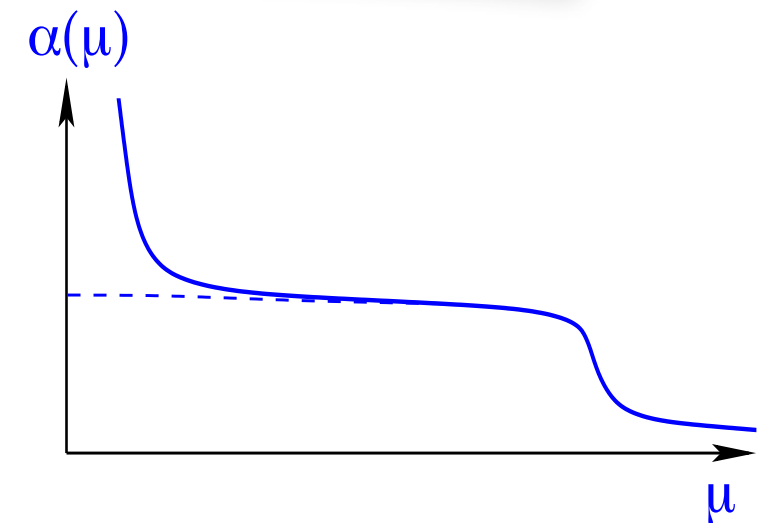
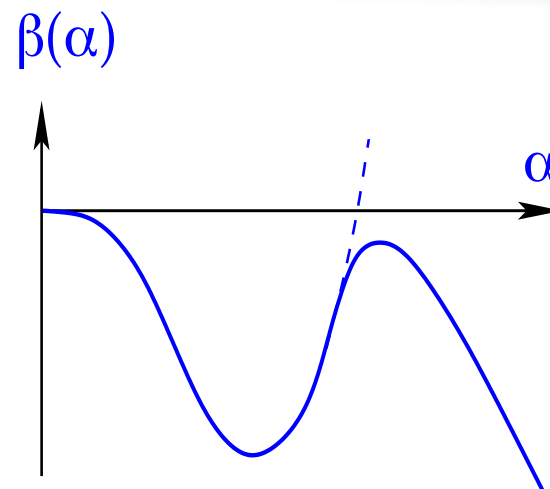
- key: to

Is it possible to construct such a theory ?

moto]

- renormalized gauge coupling

- to run very slowly (walking)



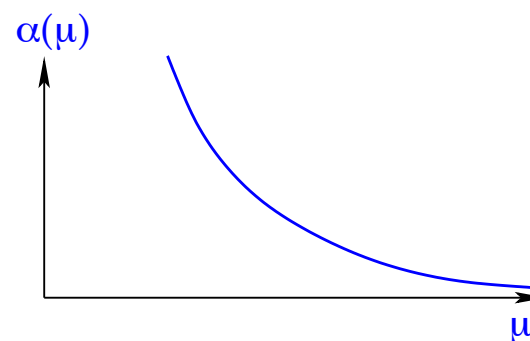
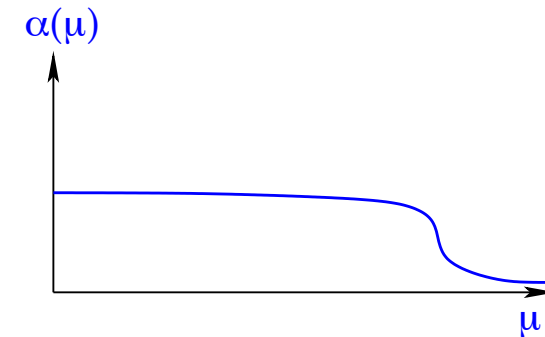
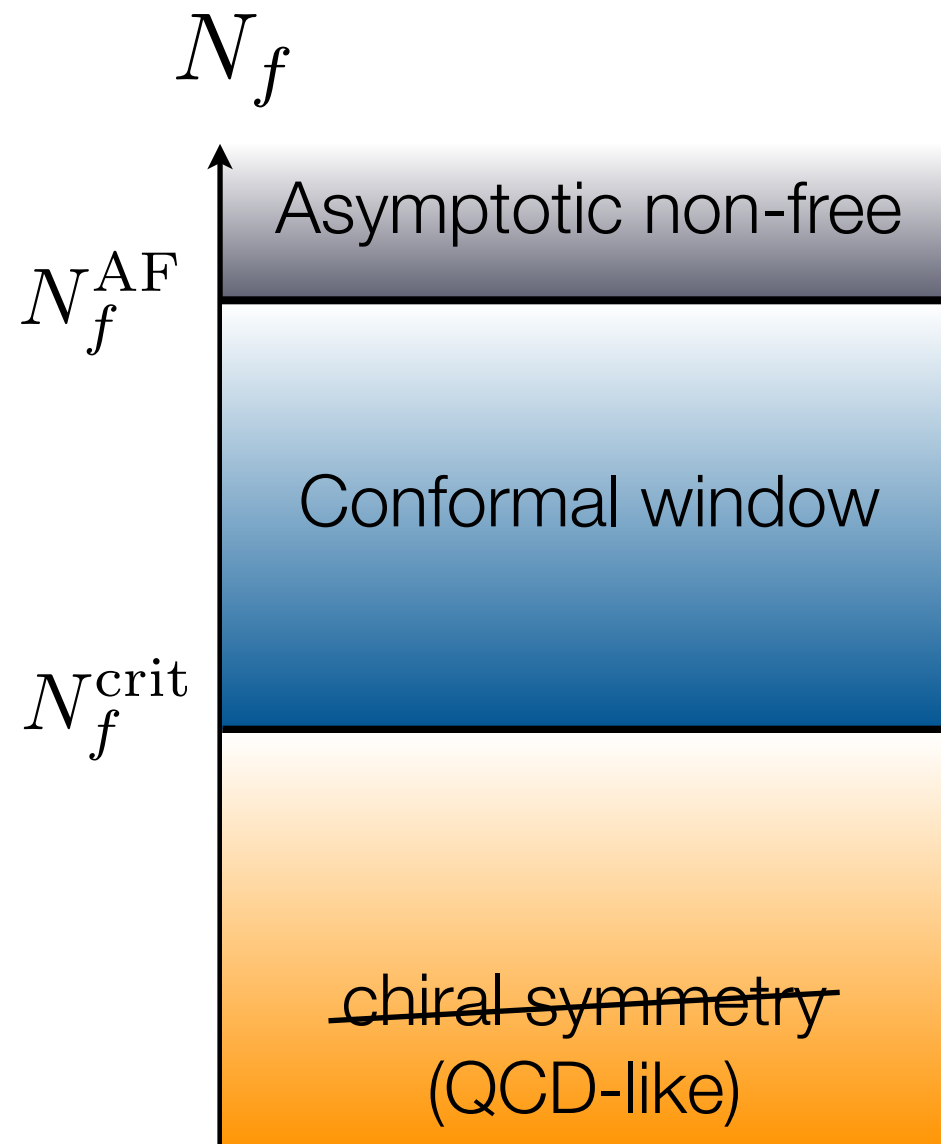
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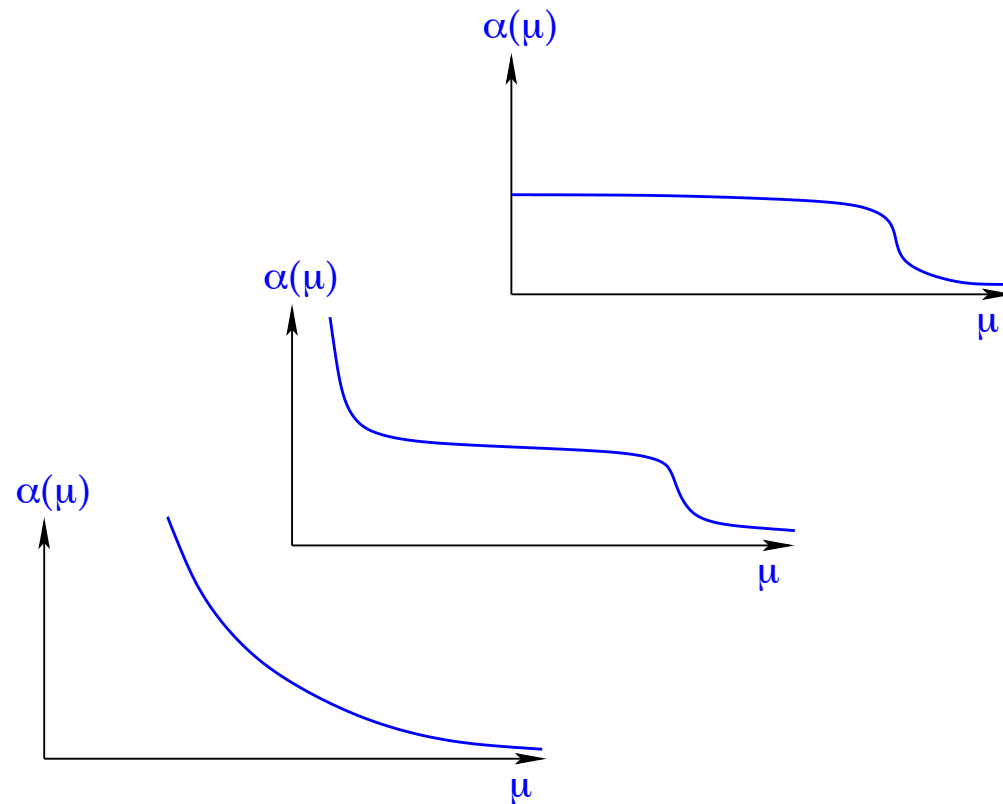
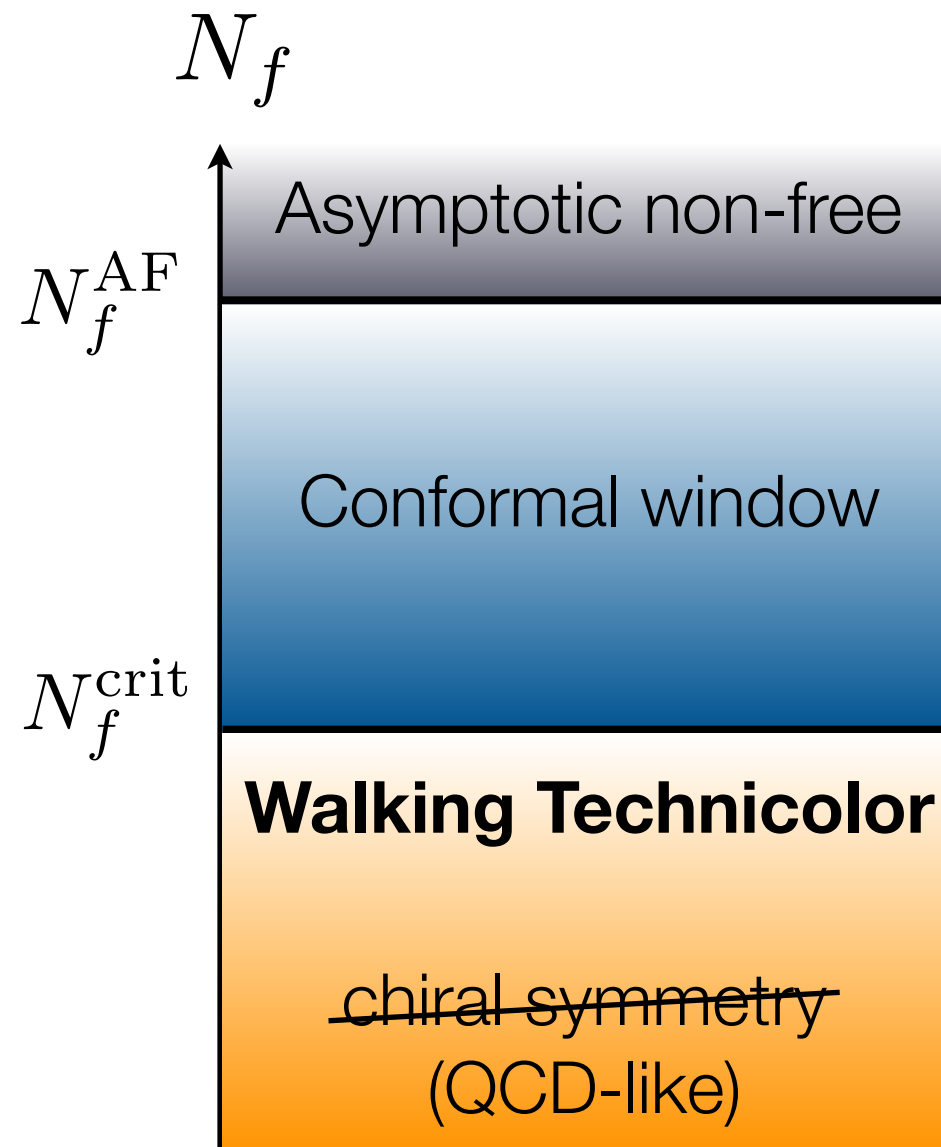
conformal window and walking gauge coupling

- non-Abelian gauge theory with N_f *massless* fermions -



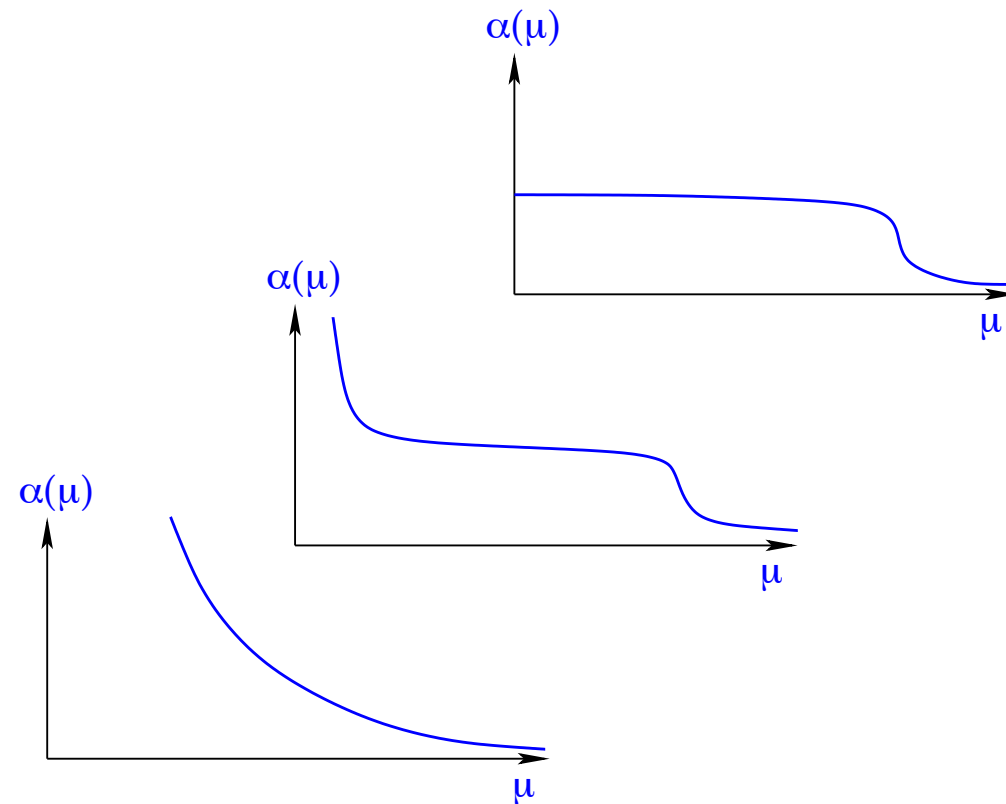
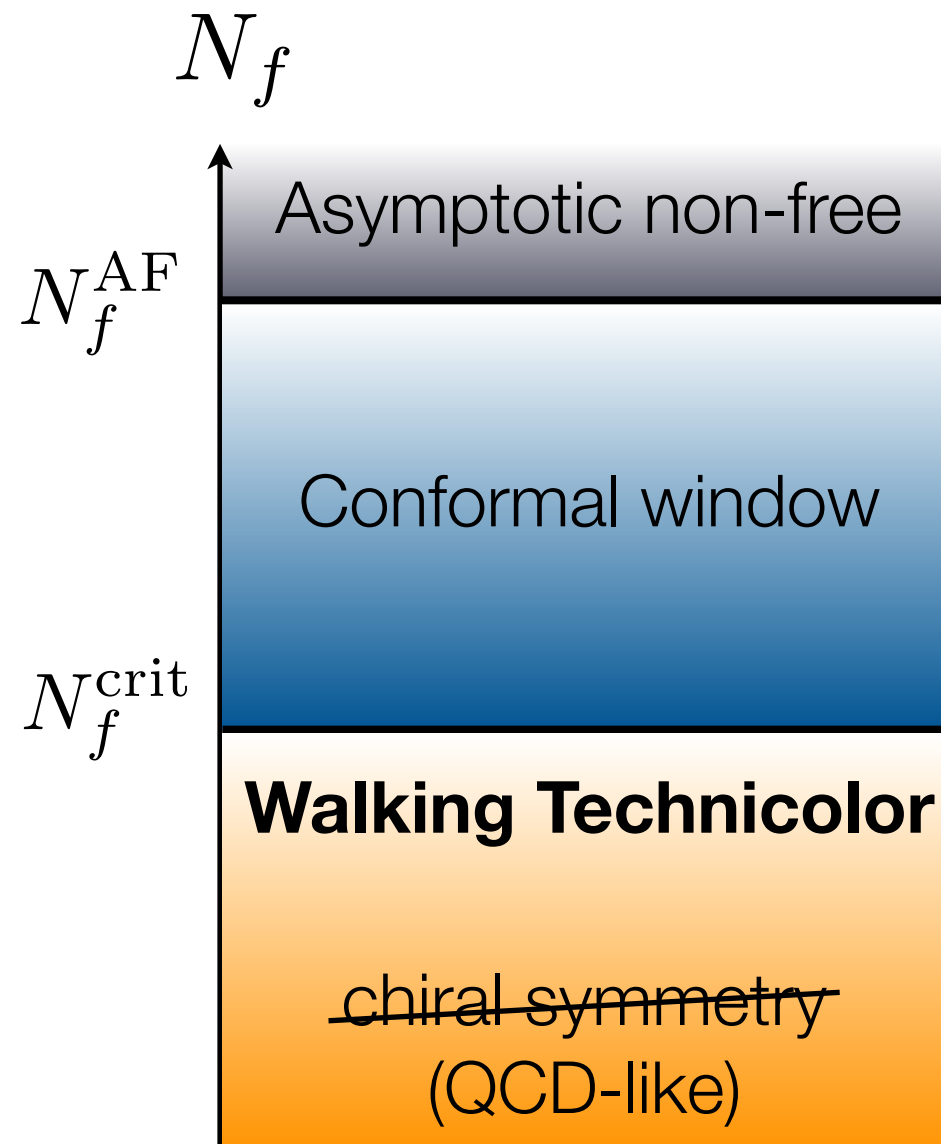
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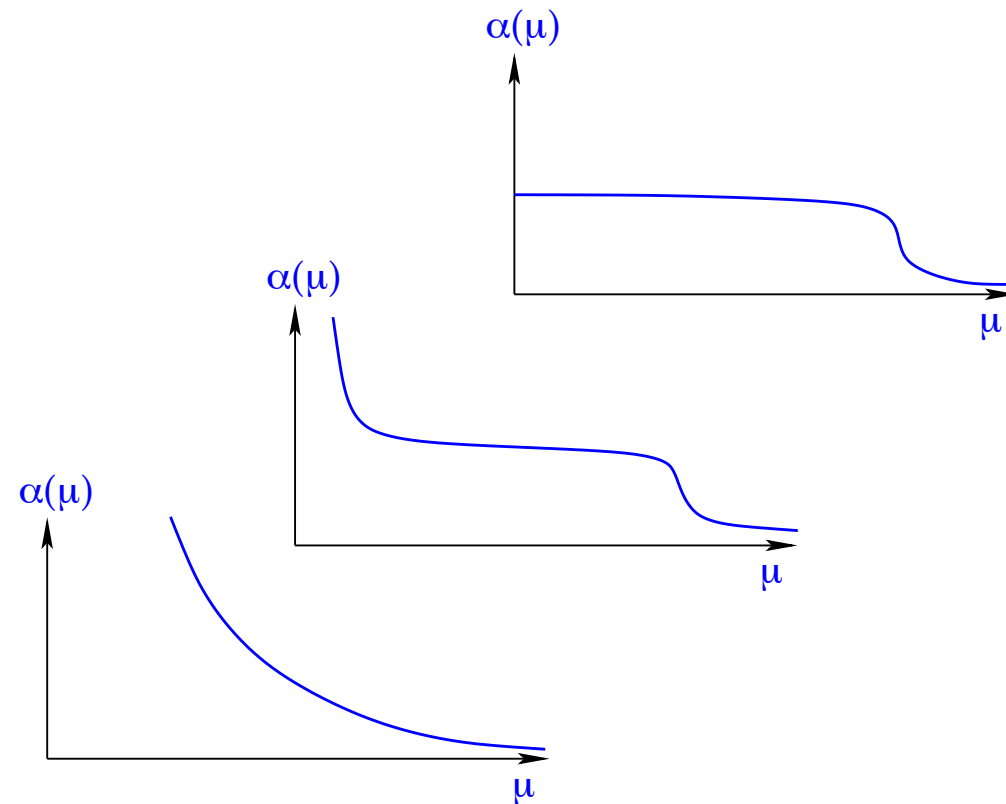
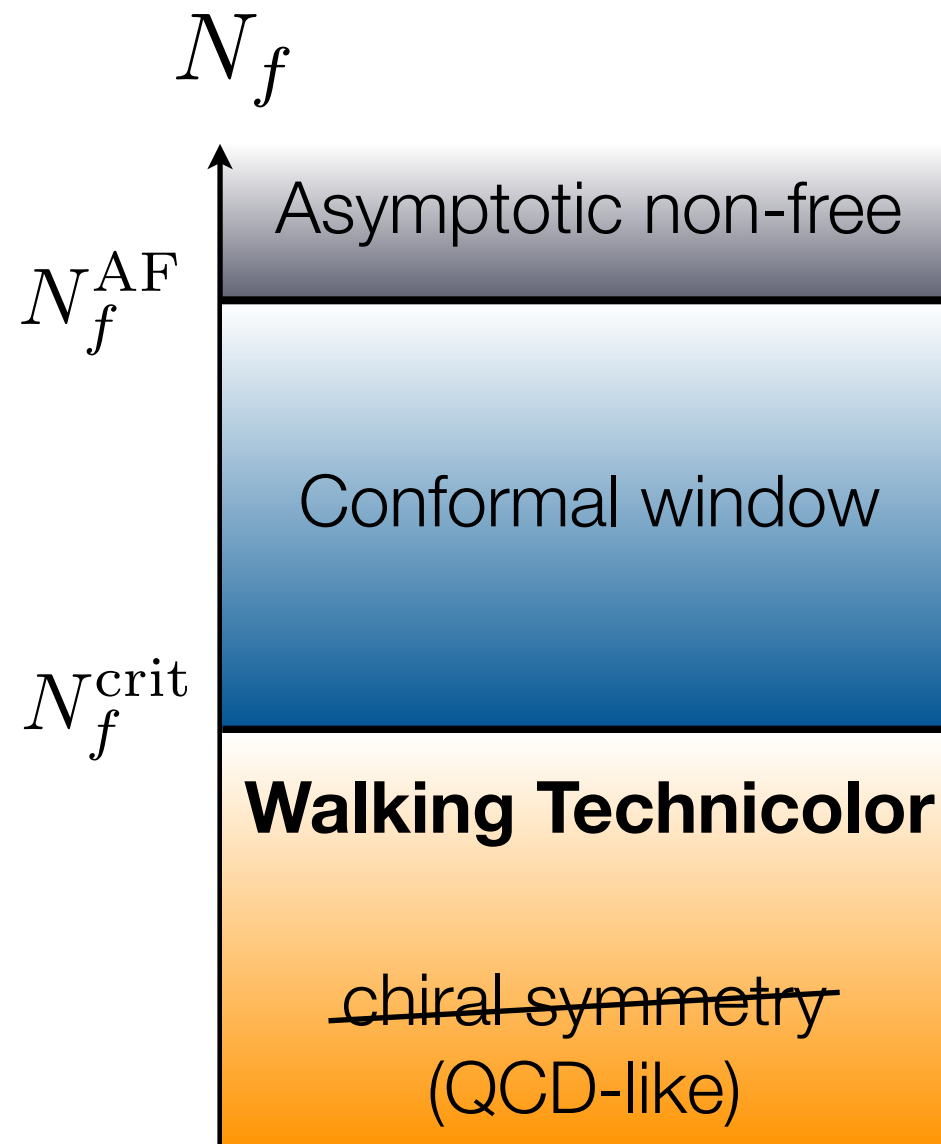
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- Walking Technicolor could be realized just below the conformal window

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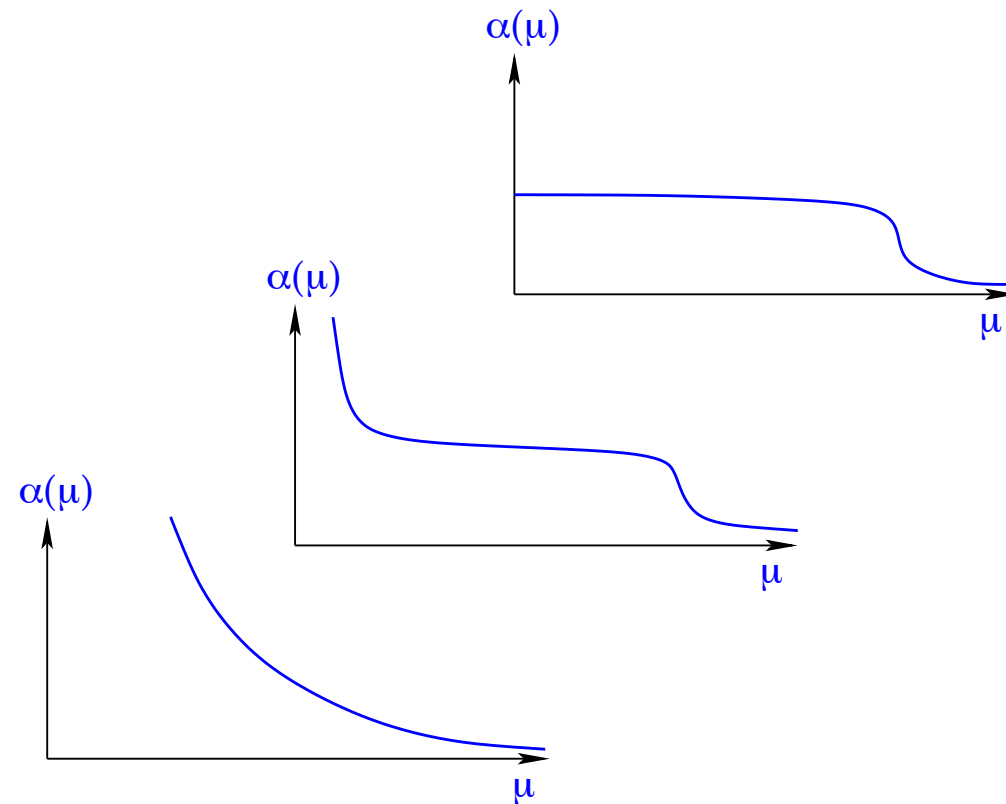
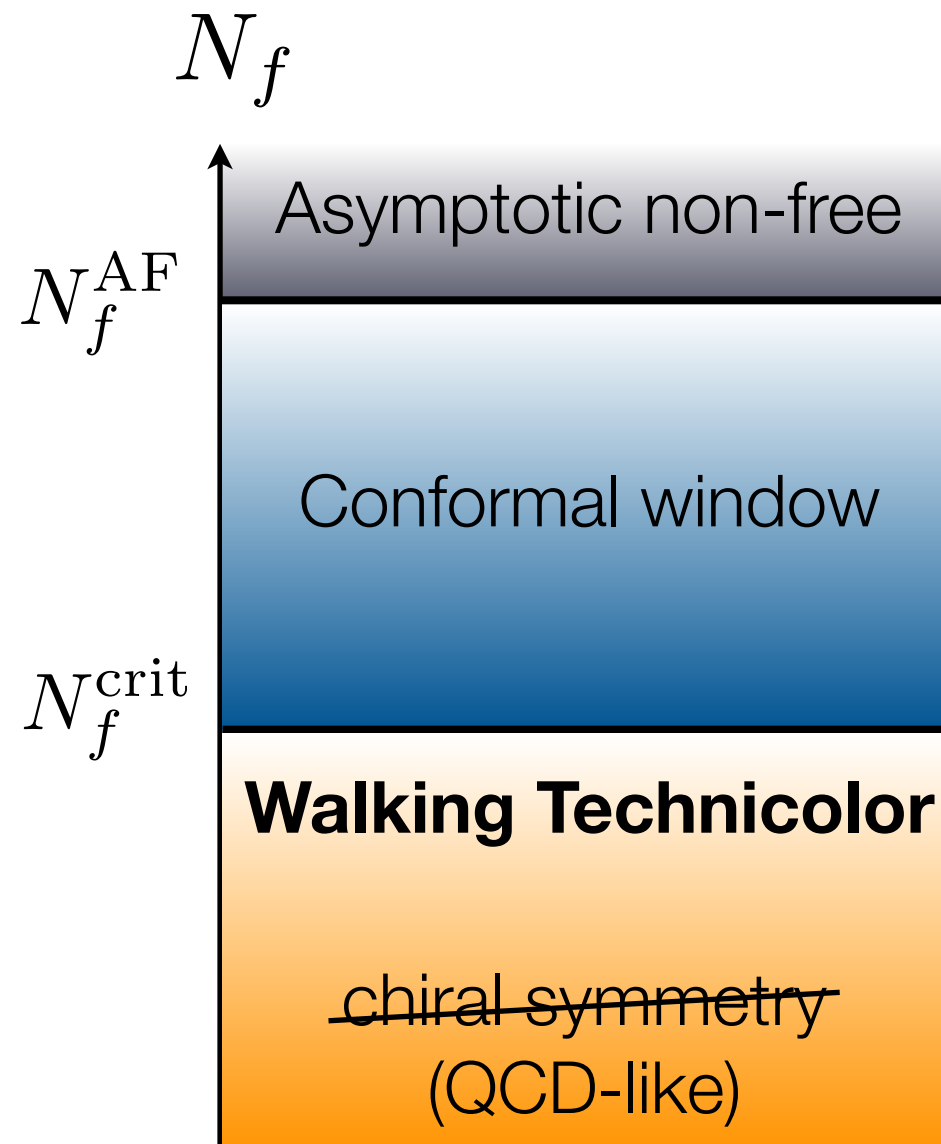
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- crucial information: N_f^{crit} and...

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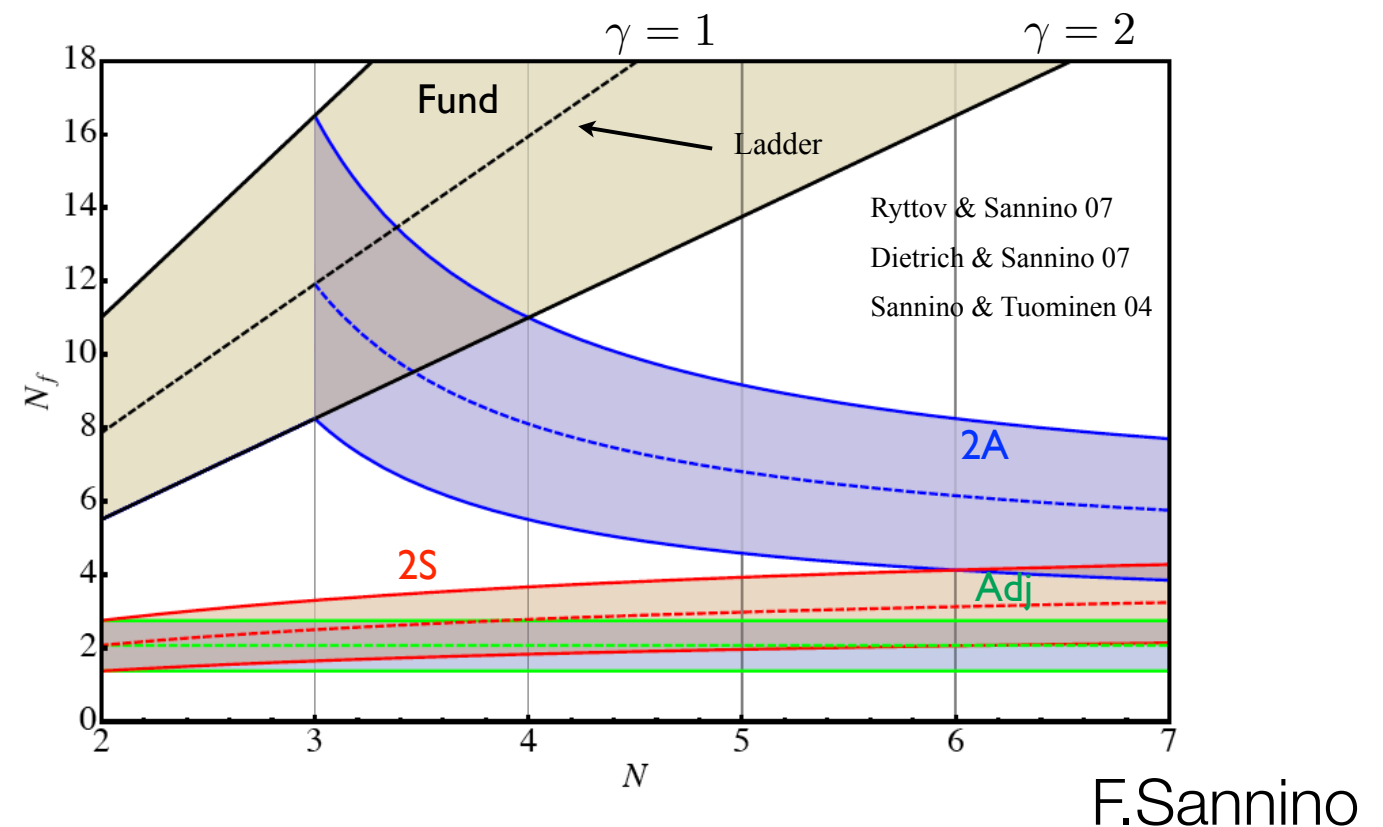


- Walking Technicolor could be realized just below the conformal window
- crucial information: N_f^{crit} and...
- mass anomalous dimension γ & the composite mass spectrum around N_f^{crit}

models being studied:

- SU(3)
 - fundamental: $N_f=6, 8, 10, 12, 16$
 - sextet: $N_f=2$
- SU(2)
 - adjoint: $N_f=2$
 - fundamental: $N_f=8$
- SU(4)
 - decuplet: $N_f=2$

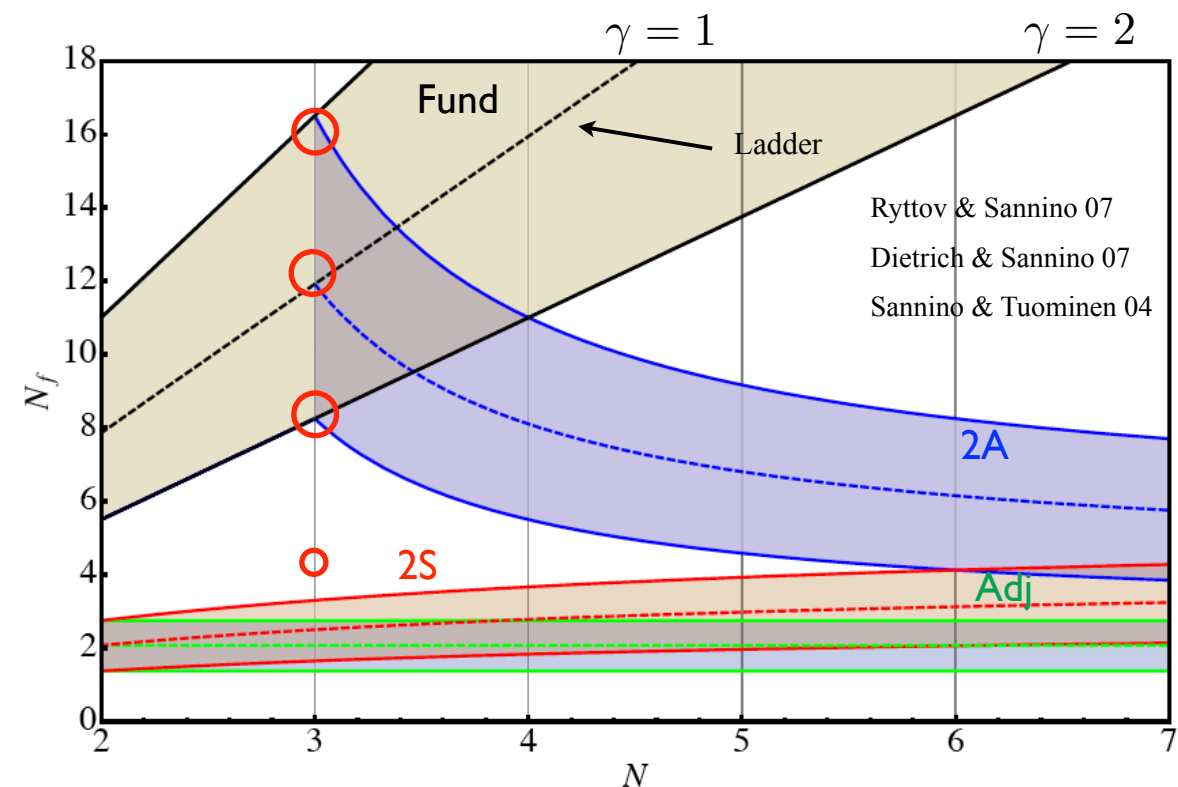
SU(N) Phase Diagram



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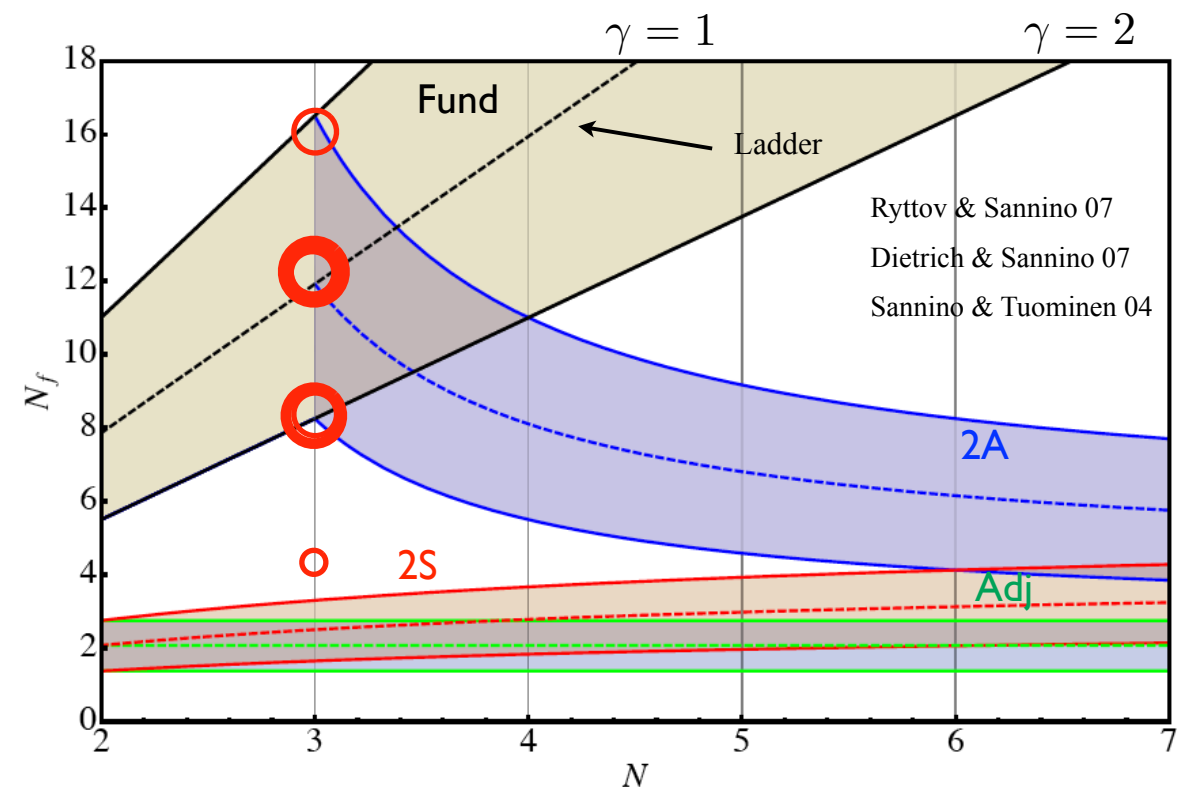
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SU(N) Phase Diagram



LatKMI collaboration

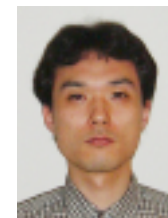
- YA, T.Aoyama, M.Kurachi, T.Maskawa, K.Miura, K.Nagai,



H.Ohki, K.Yamawaki, T.Yamazaki



名古屋大学



- E. Rinaldi



A. Shibata



LatKMI mission

- find / understand (near) **conformal dynamics** in gauge theory: late 2010
 - using a state-of-the-art lattice discretization (HISQ) and computation
- find **conformal window** in SU(3) gauge theory w. N_f $m=0$ fundamental fermions
- find a **walking technicolor theory** in SU(3) gauge theory
- investigate $N_f=8$ in some detail
- investigate **flavor singlet scalar** in SU(3) gauge theory
- test $N_f=8$ against experiment

LatKMI publications

- LatKMI, PRD 85 (2012), “Study of the conformal hyperscaling relation through the Schwinger-Dyson equation” [non-lattice]
- LatKMI, PRD 86 (2012), “Lattice study of conformality in twelve-flavor QCD”
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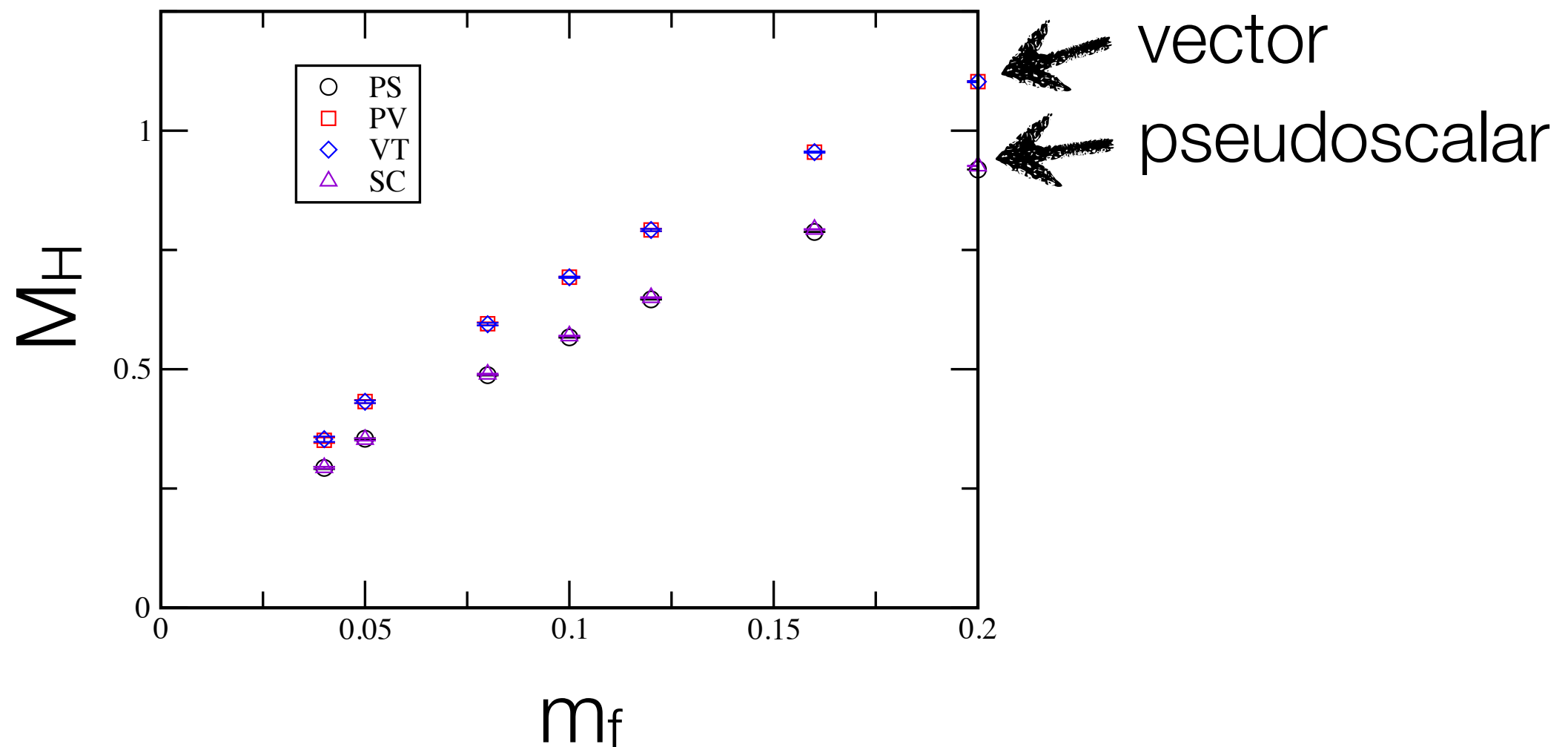
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Simulation

- Fermion Formulation: HISQ (Highly Improved Staggered Quarks)
 - being used for state-of-the-art QCD calculations / MILC,..
- Gauge Field Formulation: tree level Symanzik gauge
- $N_f=4$: $\beta=6/g^2=3.7$, $V=L^3 \times T$: $L/T=2/3$; $L=12, 16$
- $N_f=8$: $\beta=6/g^2=3.8$, $V=L^3 \times T$: $L/T=3/4$; $L=18, 24, 30, 36$
- $N_f=12$ (two lattice spacings):
 - $\beta=6/g^2=3.7$, $V=L^3 \times T$: $L/T=3/4$; $L=18, 24, 30$, $0.04 \leq m_f \leq 0.2$
 - $\beta=6/g^2=4.0$, $V=L^3 \times T$: $L/T=3/4$; $L=18, 24, 30$, $0.05 \leq m_f \leq 0.24$
- using MILC code v7, with modification: HMC and speed up in MD

staggered flavor symmetry for $N_f=12$ HISQ

- comparing masses with different staggered operators for π & ρ for $\beta=3.7$



- excellent staggered flavor symmetry, thanks to HISQ

Hadron spectrum:

m_f -response in mass deformed theory

- IR conformal phase:
 - coupling runs for $\mu < m_f$: like $n_f=0$ QCD with $\Lambda_{\text{QCD}} \sim m_f$
 - multi particle state : $M_H \propto m_f^{1/(1+\gamma_m^*)}$; $F_\pi \propto m_f^{1/(1+\gamma_m^*)}$ (criticality @ IRFP)
- S χ SB phase:
 - ChPT
 - at leading: $M_\pi^2 \propto m_f$, ; $F_\pi = F + c m_f$

a crude study using ratios

- conformal scenario:

- $M_H \propto m_f^{1/(1+\gamma_m^*)}$; $F_\pi \propto m_f^{1/(1+\gamma_m^*)}$ for small m_f

- ★ $F_\pi/M_\pi \rightarrow \text{const.}$ for small m_f

- ★ $M_\rho/M_\pi \rightarrow \text{const.}$ for small m_f

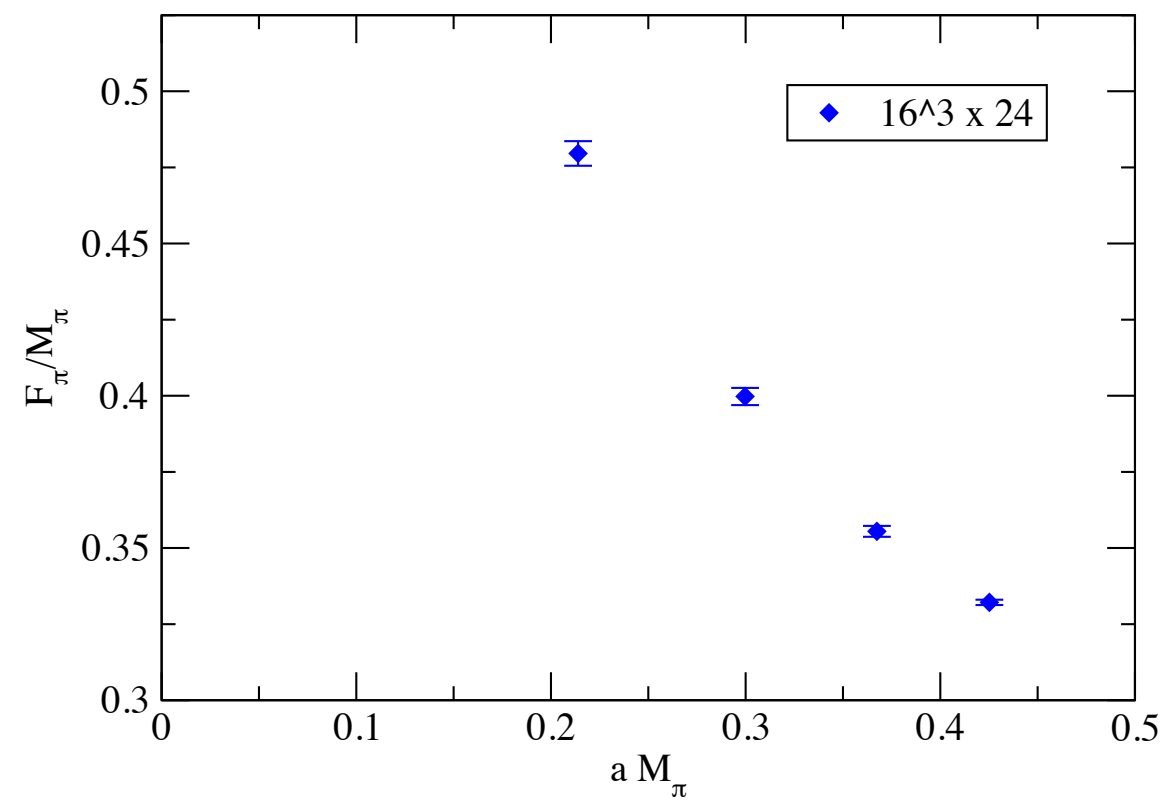
- chiral symmetry breaking scenario:

- $M_\pi^2 \propto m_f$, ; $F_\pi = F + c' M_\pi^2$ for small m_f

- ★ $F_\pi/M_\pi \rightarrow \infty$ for $m_f \rightarrow 0$

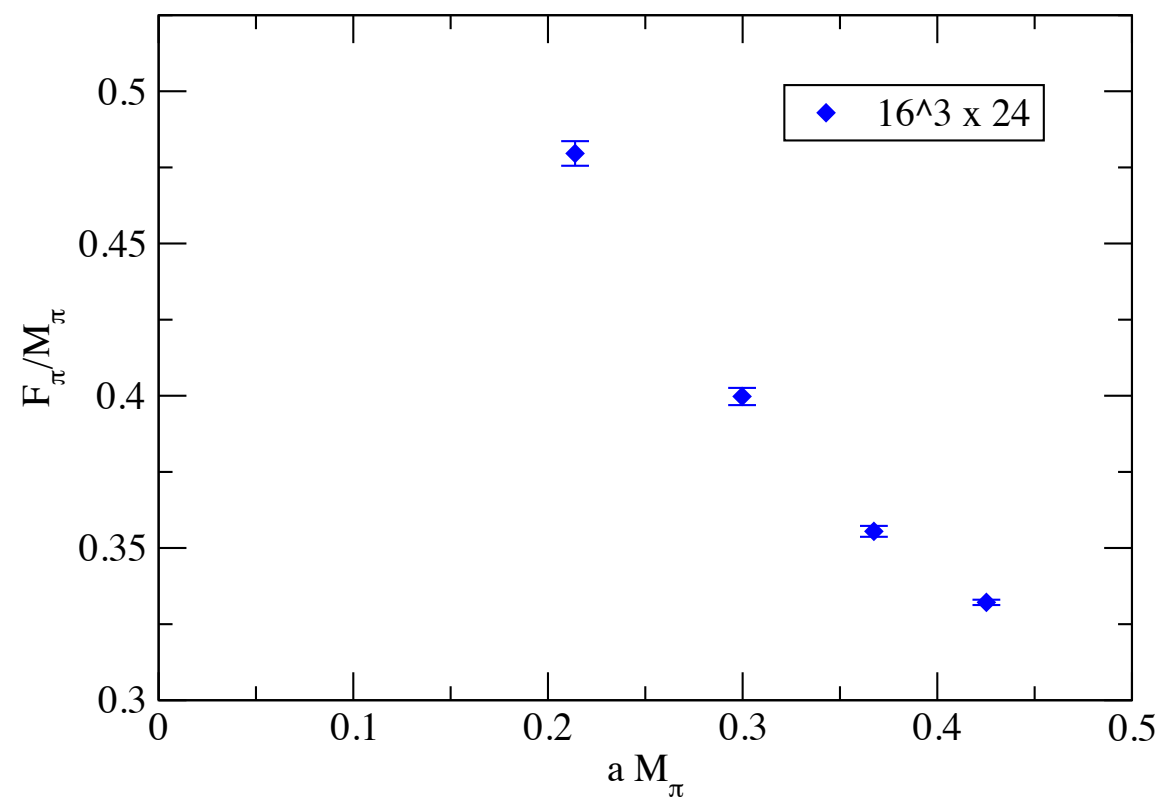
a crude analysis: F_π/M_π vs M_π

$N_f=4$



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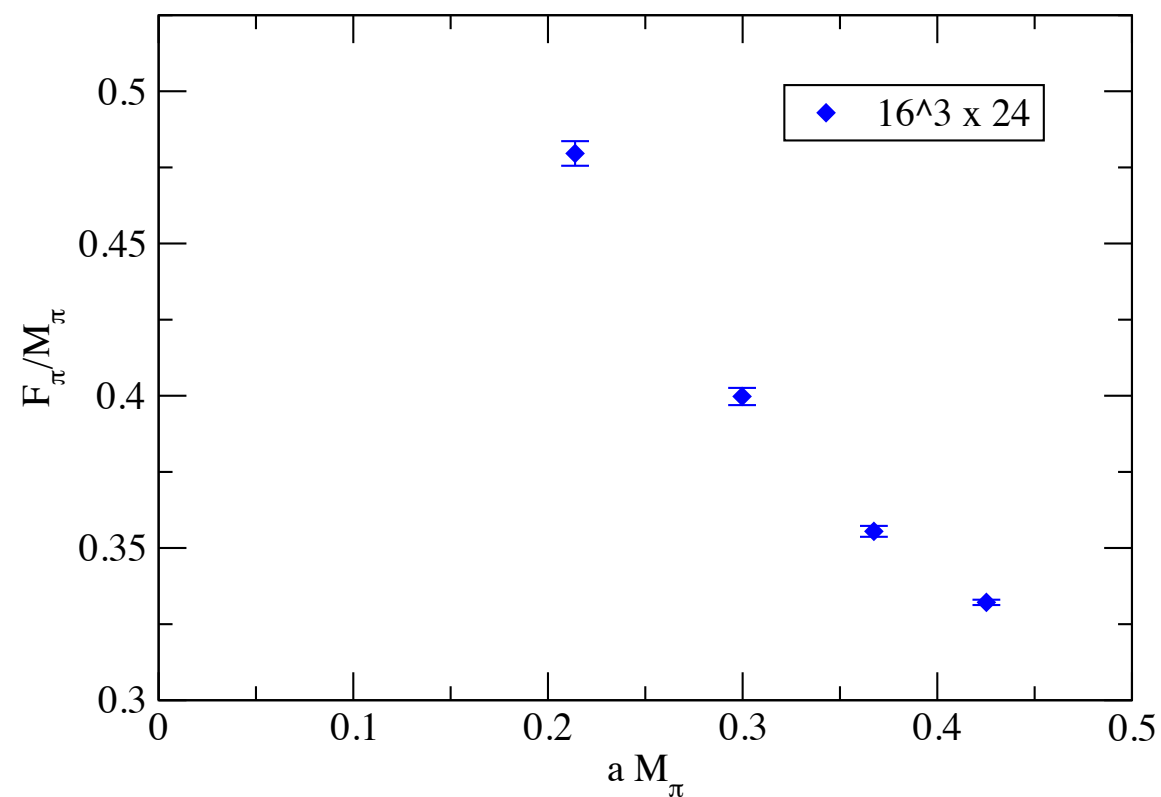
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- tends to diverge towards the chiral limit ($M_\pi \rightarrow 0$)

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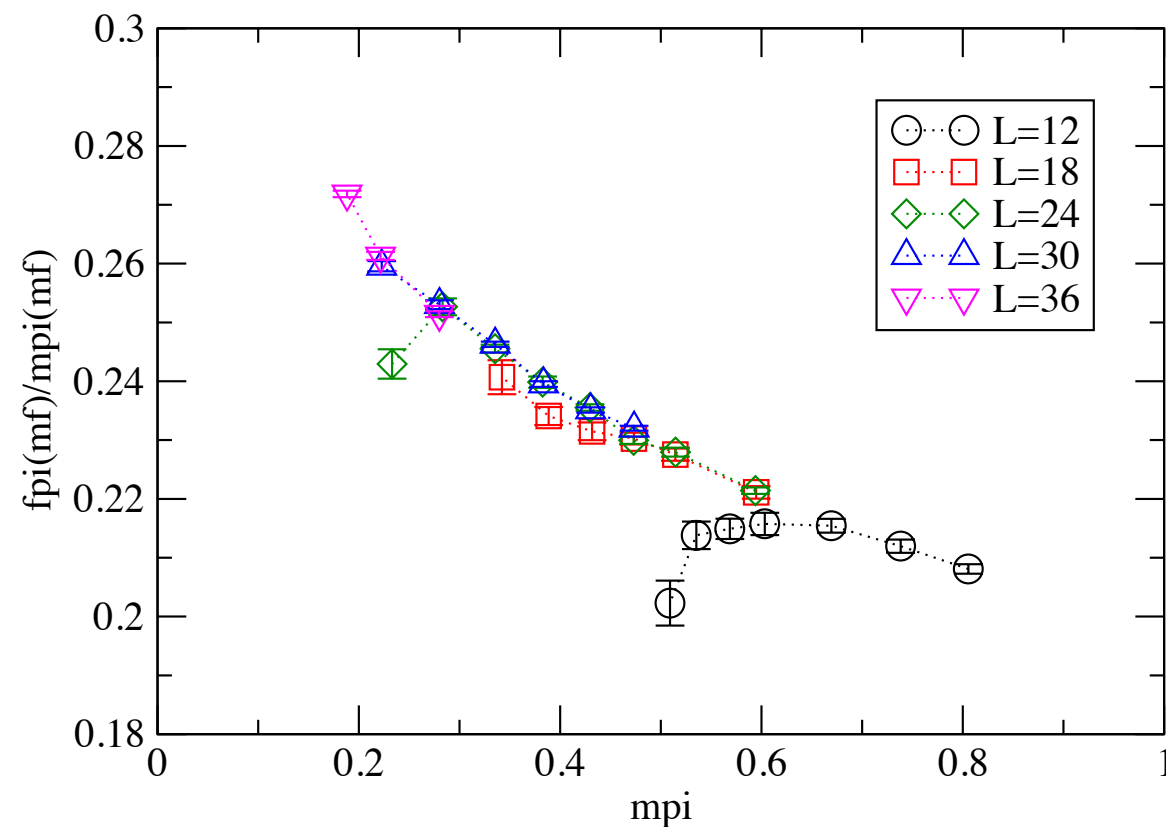
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- tends to diverge towards the chiral limit ($M_\pi \rightarrow 0$)
- spontaneous chiral symmetry breaking

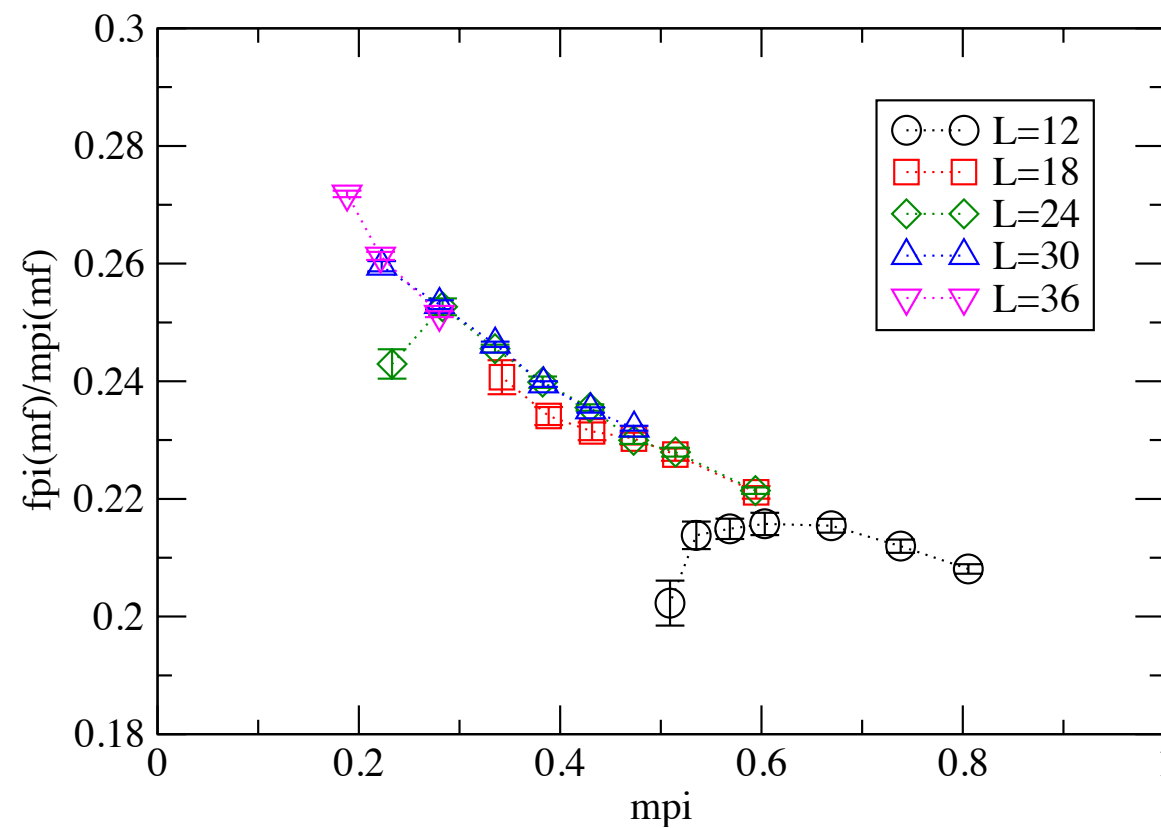
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$N_f=8$



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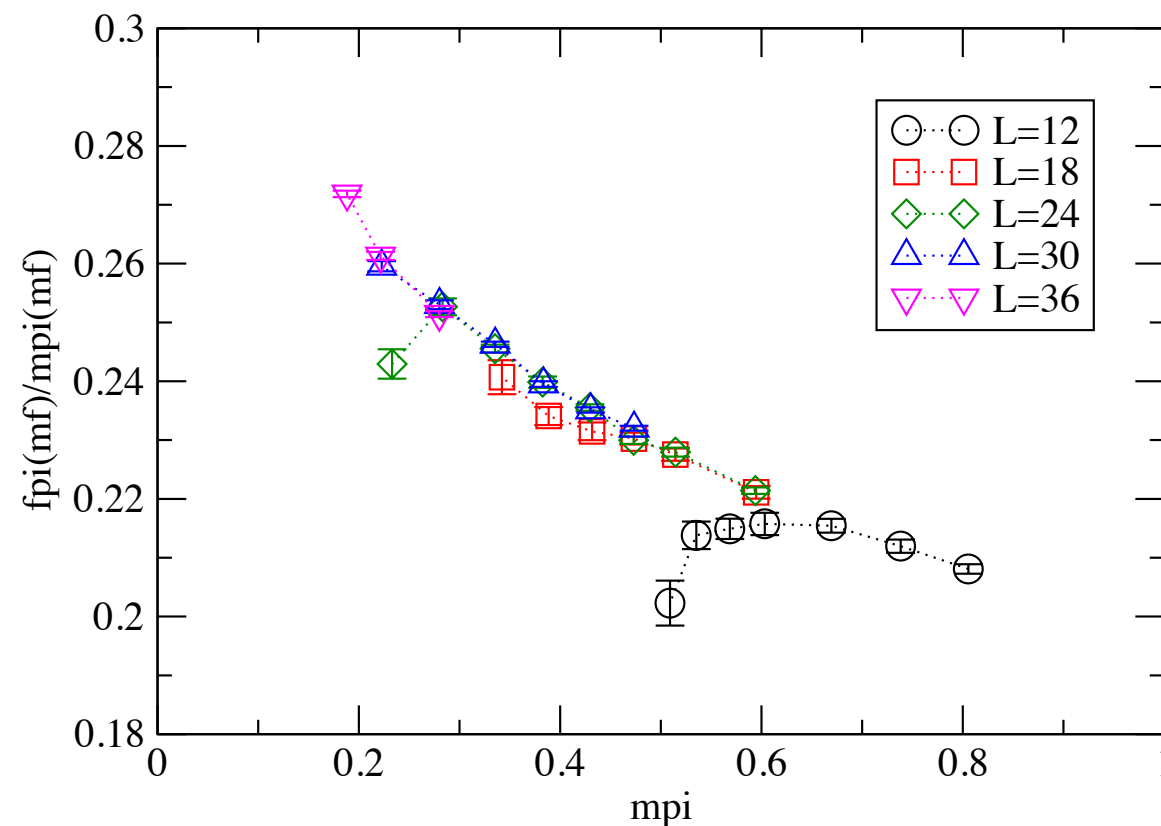
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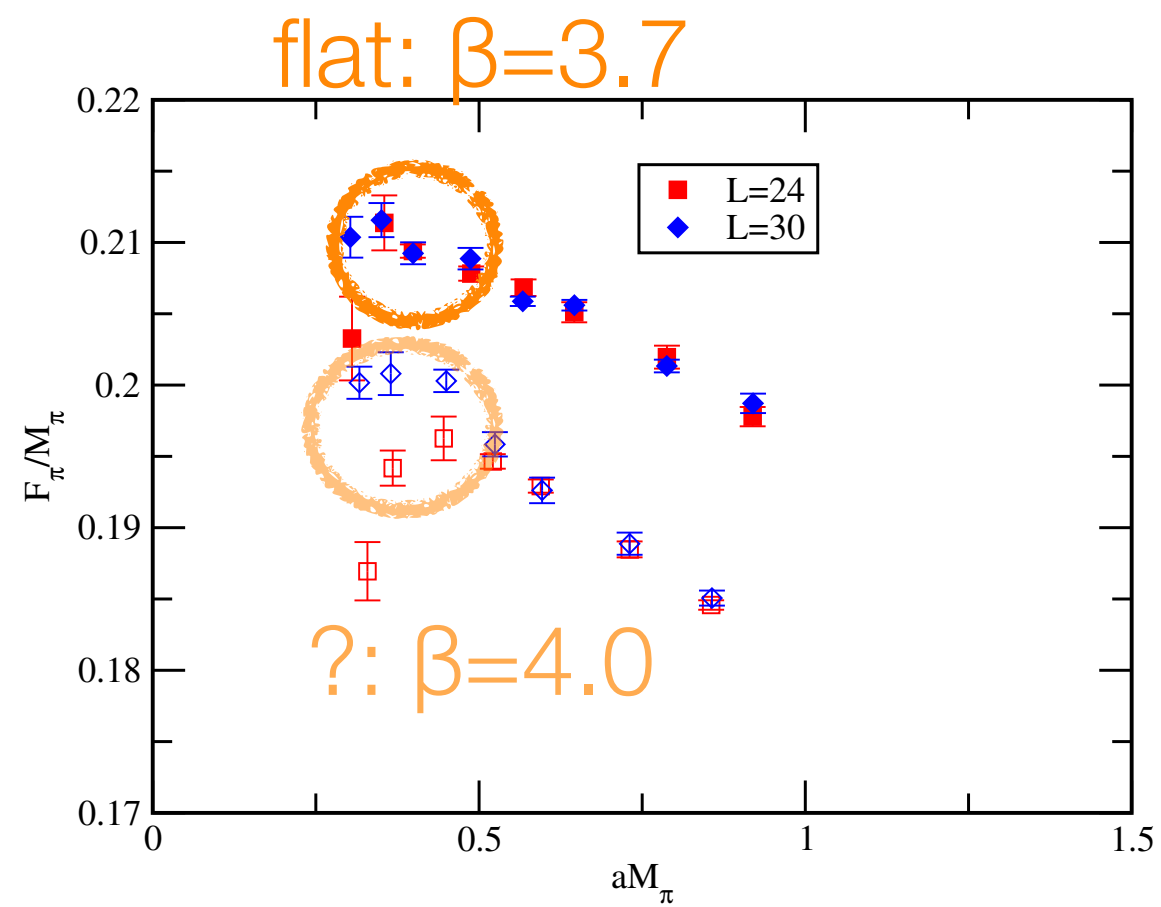
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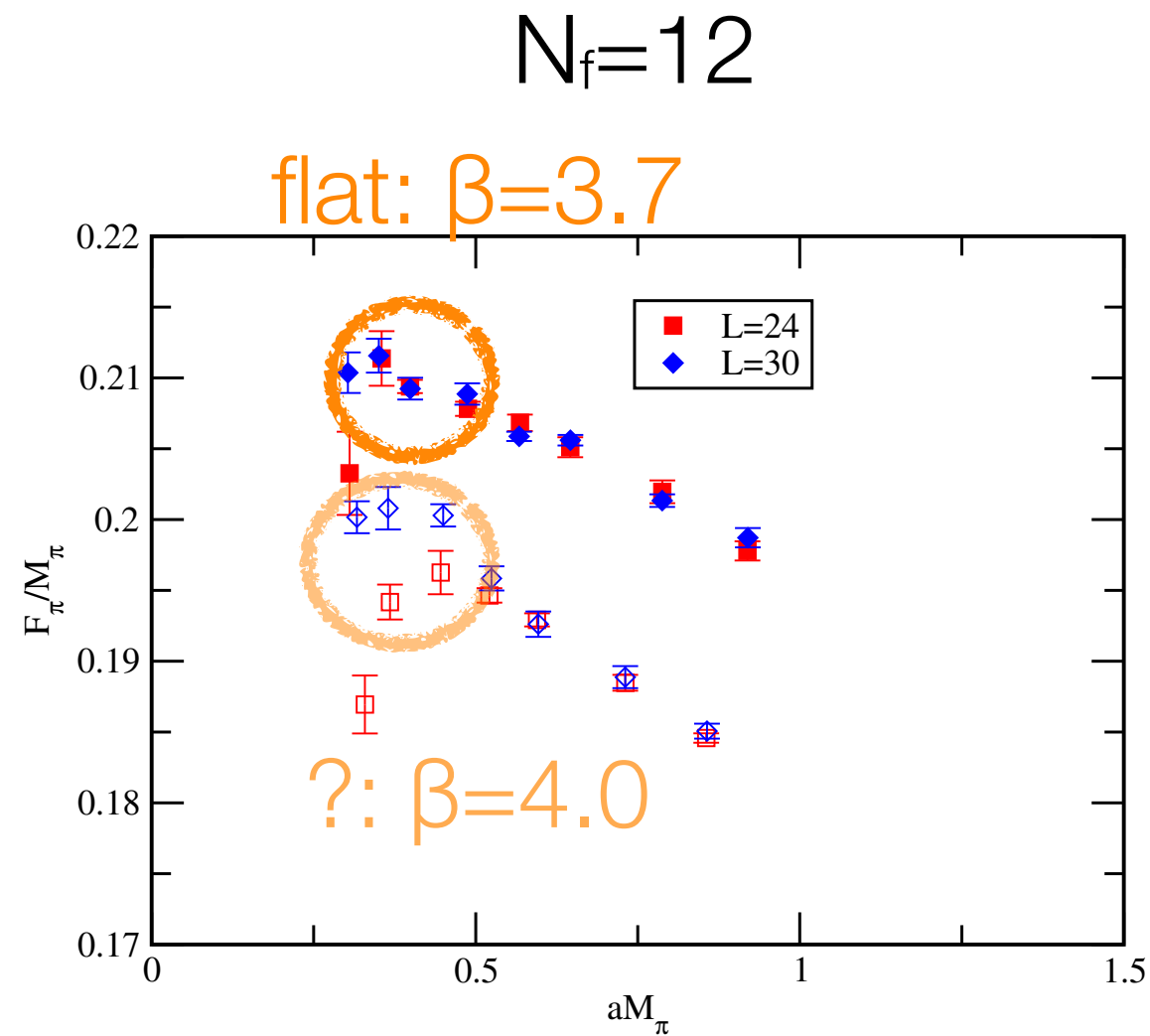
- tends to diverge towards the chiral limit ($M_\pi \rightarrow 0$)
- spontaneous chiral symmetry breaking, likely

a crude analysis: F_π/M_π vs M_π

$N_f=12$



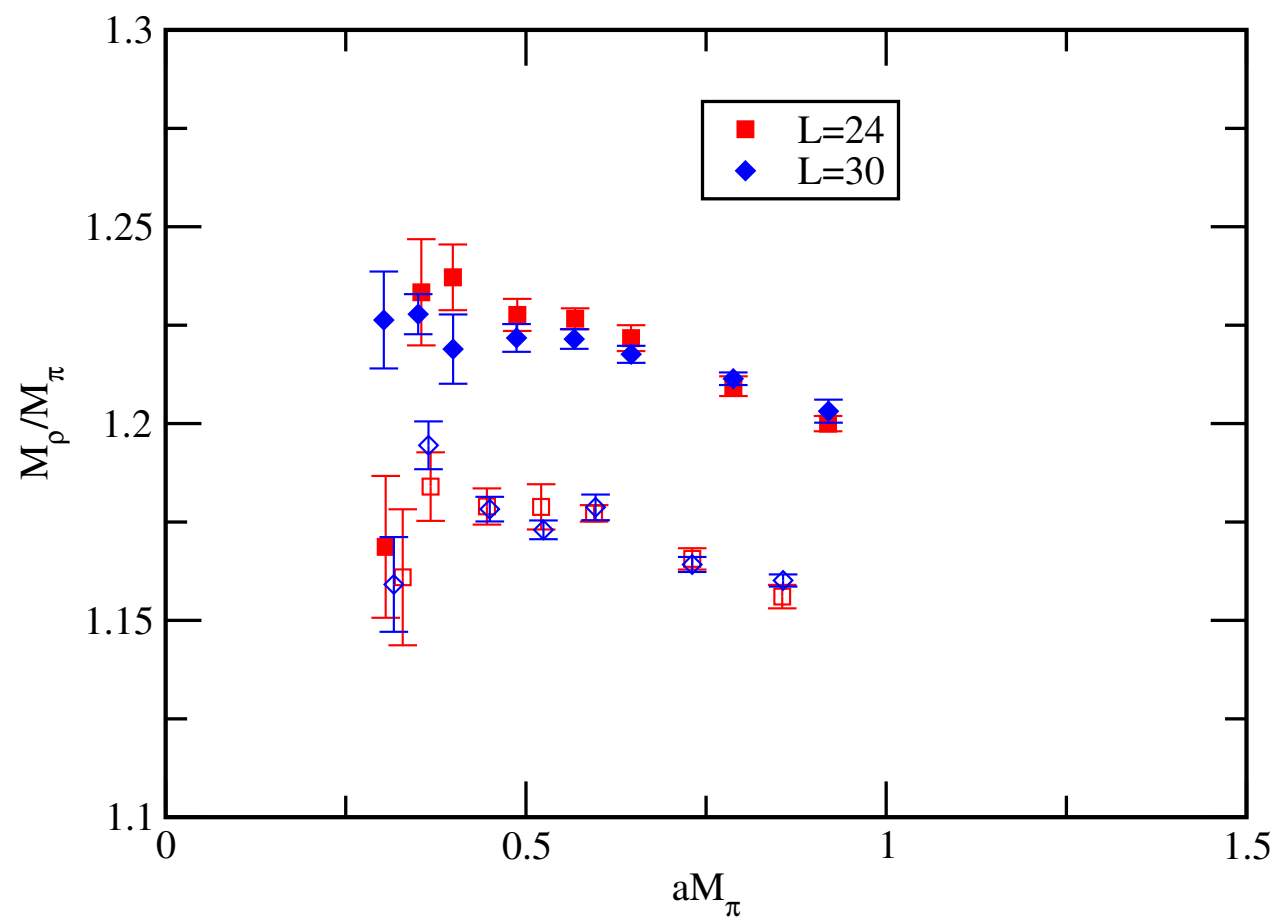
a crude analysis: F_π/M_π vs M_π



- $\beta=3.7$: small mass: consistent with conformal scenario
- $\beta=4.0$: volume likely too small to discuss the scaling

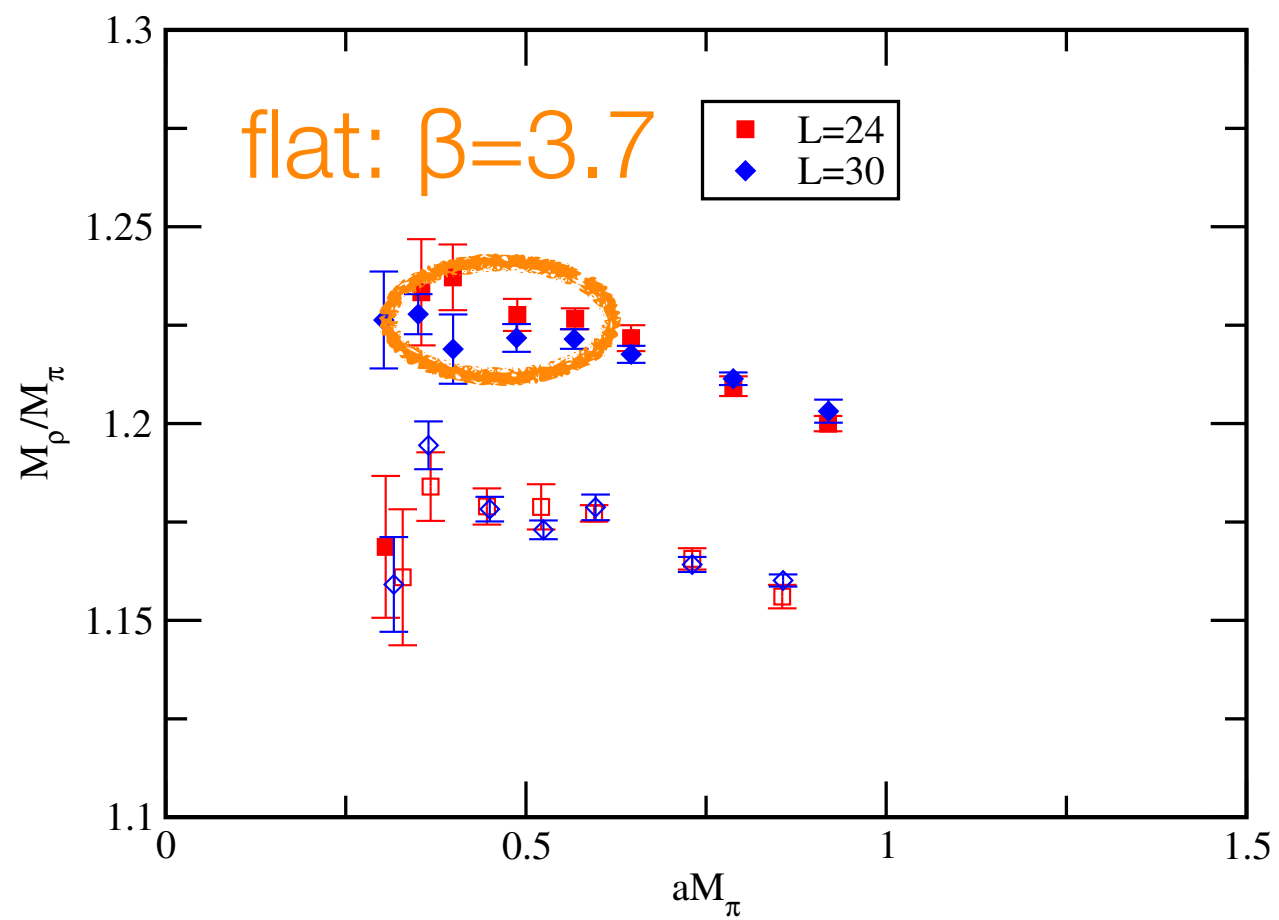
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$N_f=12$: HISQ



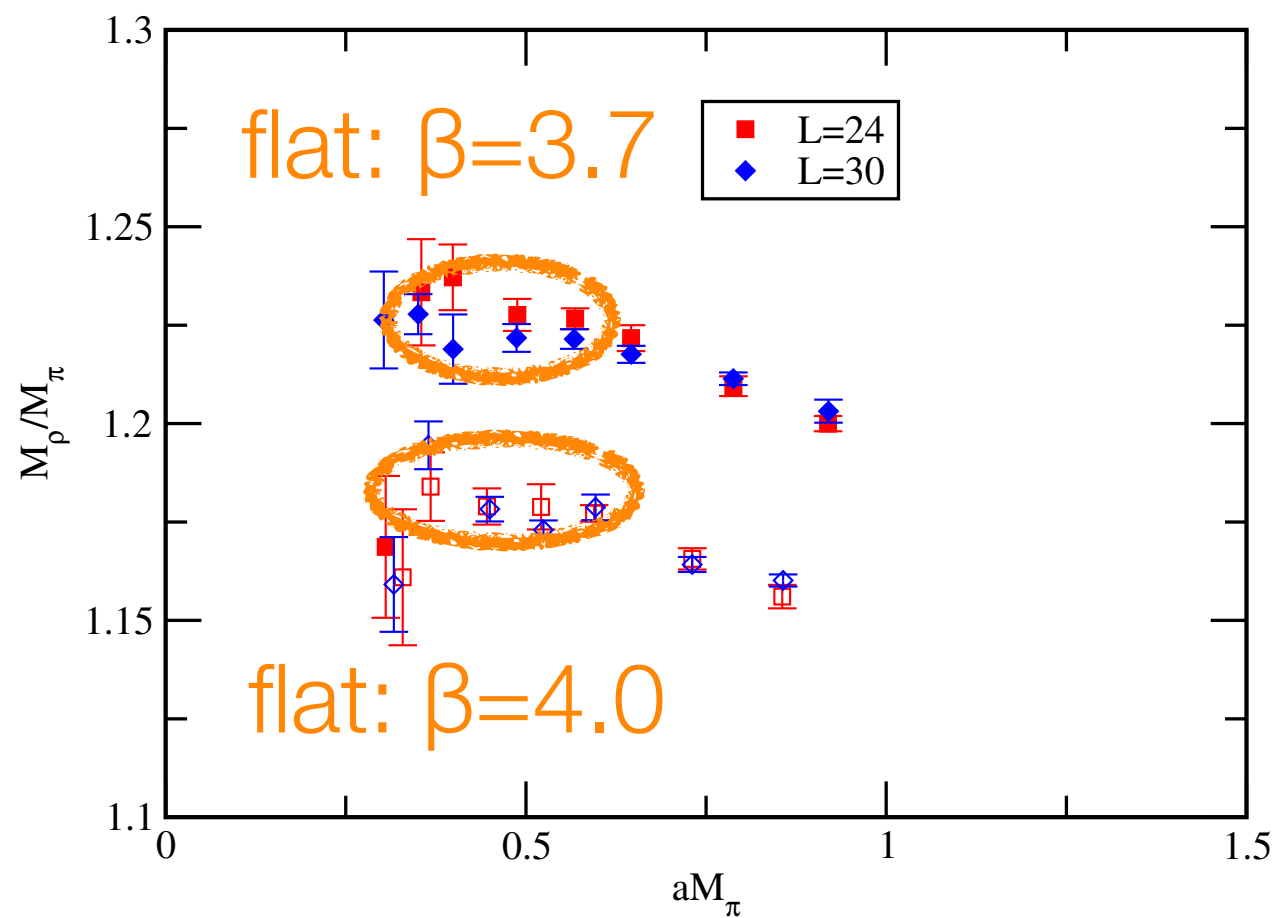
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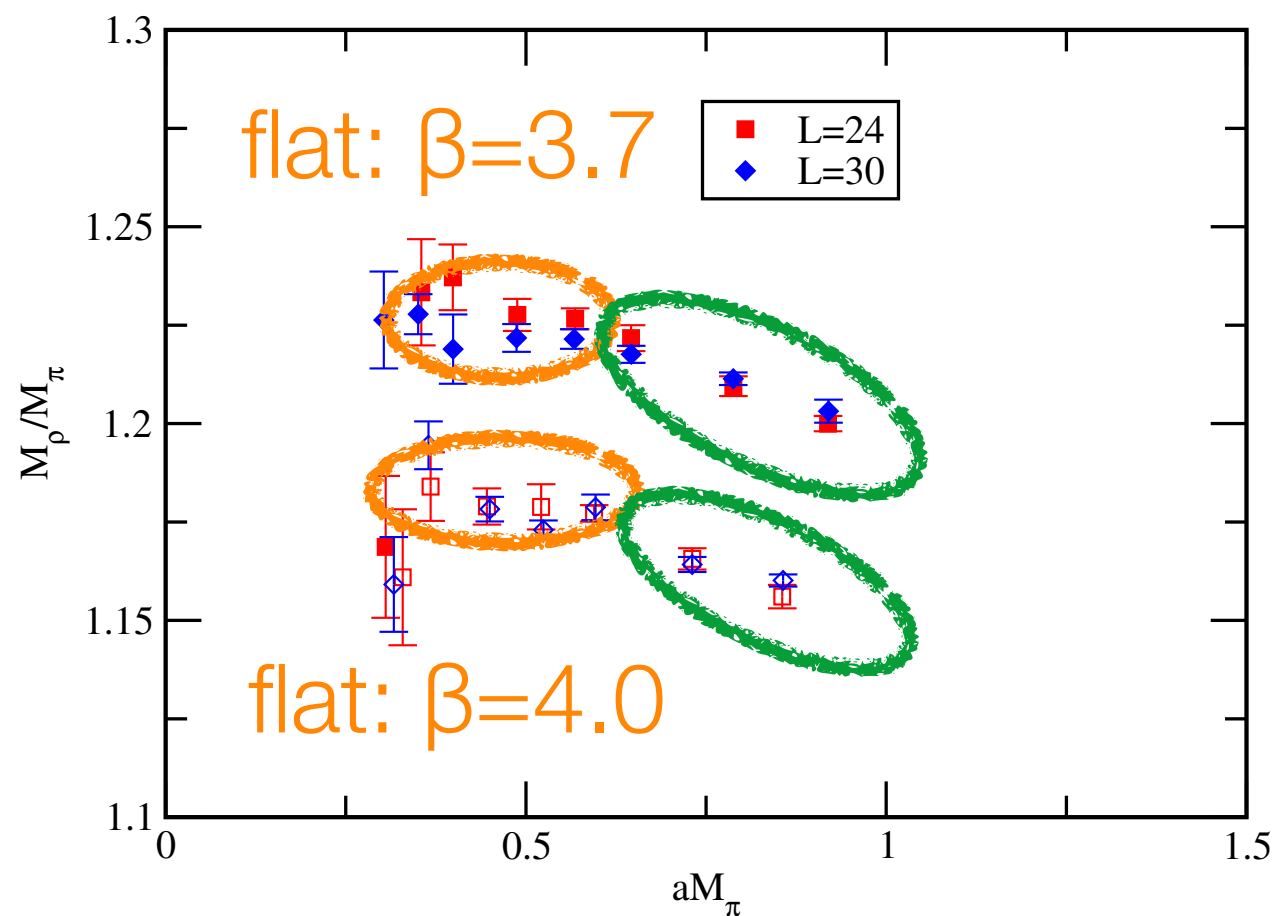
$N_f=12$: HISQ



- $\beta=3.7$ & 4.0 : small mass (wider than F_π): consistent with hyper scaling (HS)

a crude analysis: M_ρ/M_π vs M_π

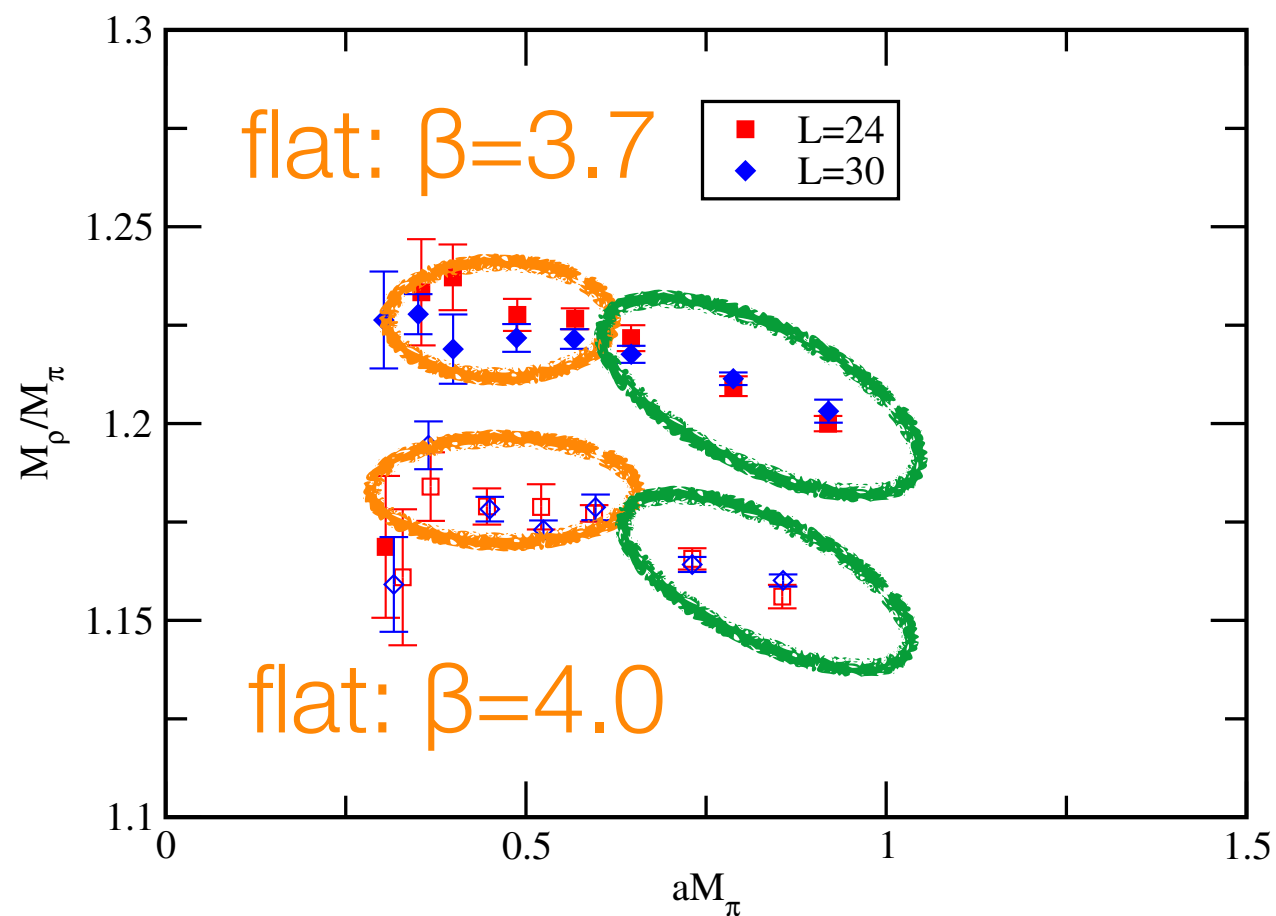
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- $\beta=3.7$ & 4.0 : small mass (wider than F_π): consistent with hyper scaling (HS)
- mass dependence at the tail is due to non-universal mass correction to HS

a crude analysis: M_ρ/M_π vs M_π

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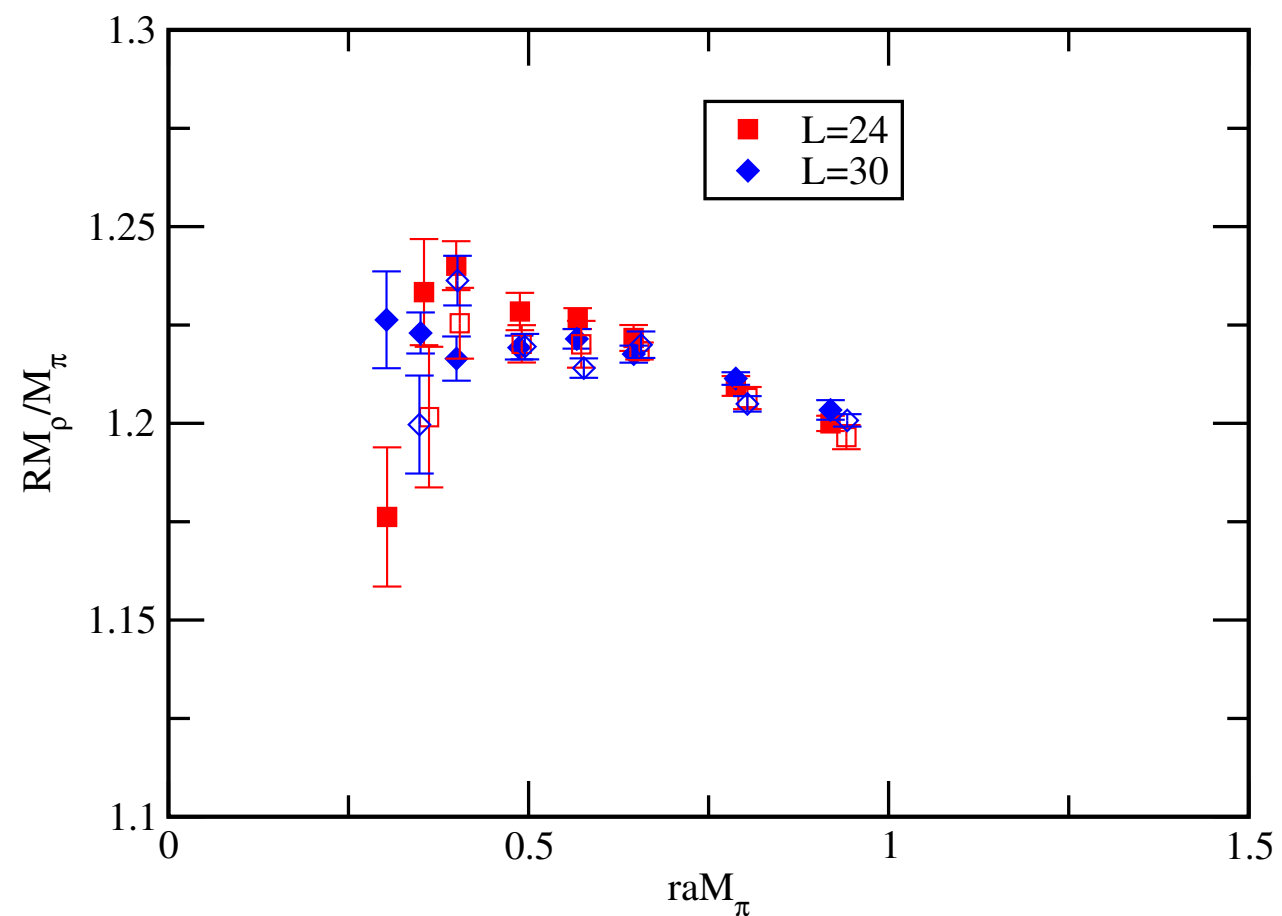


- one may attempt to perform a matching

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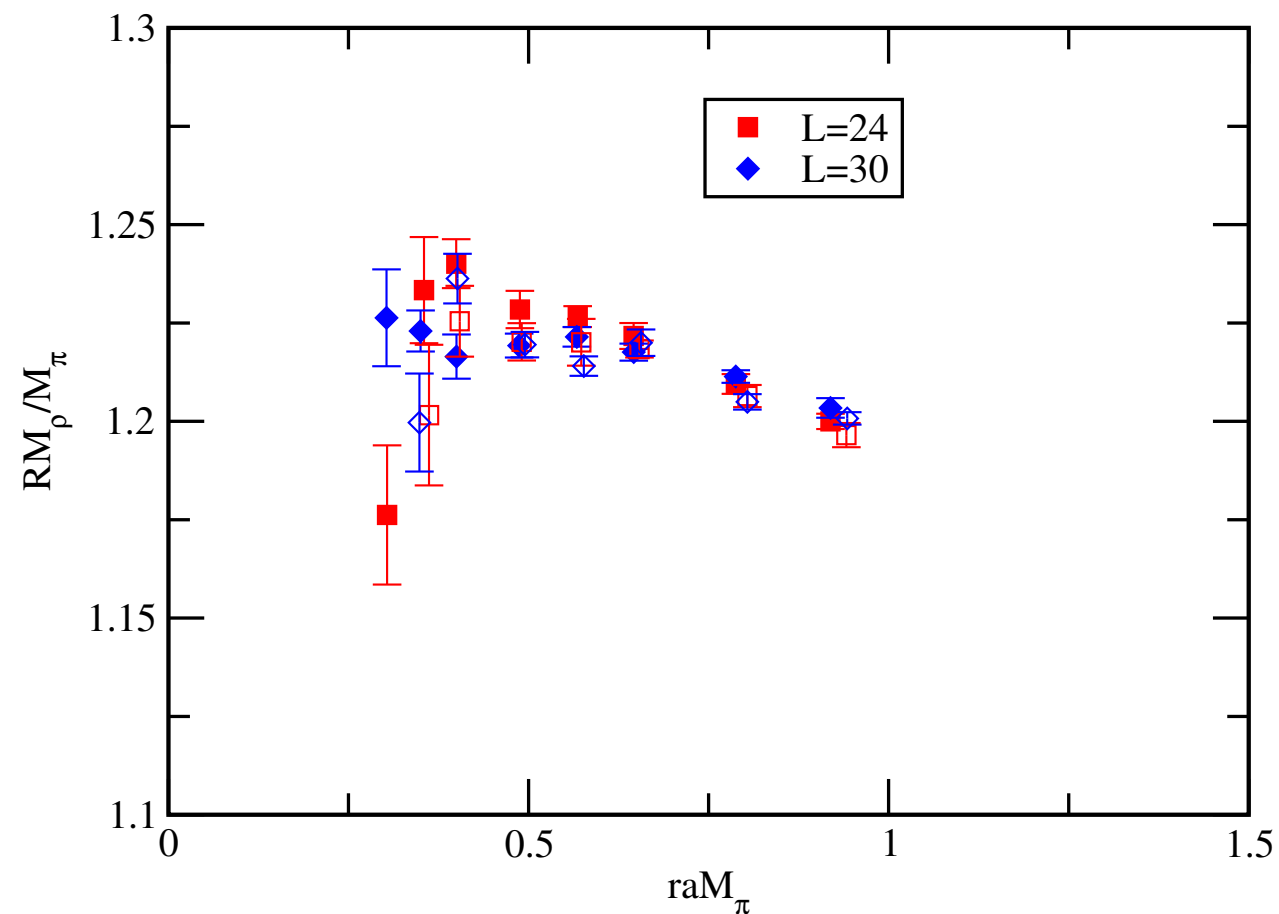


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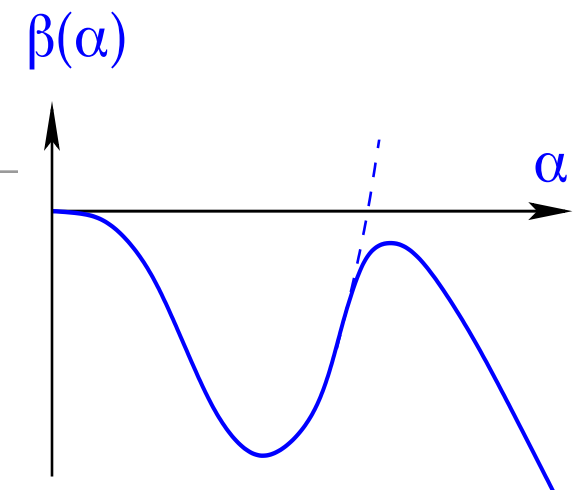
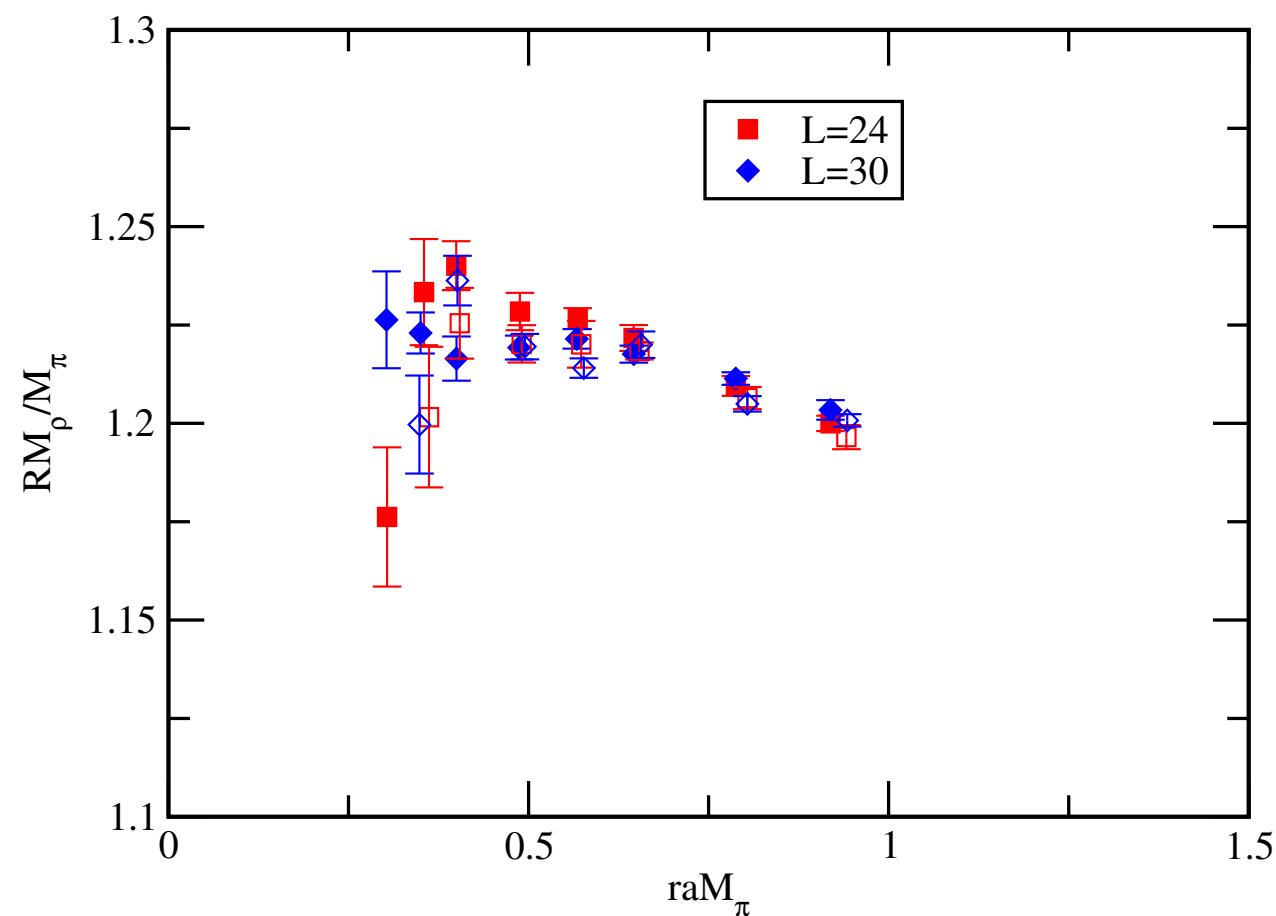
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➔ $a(\beta=3.7) / a(\beta=4.0) > 1$

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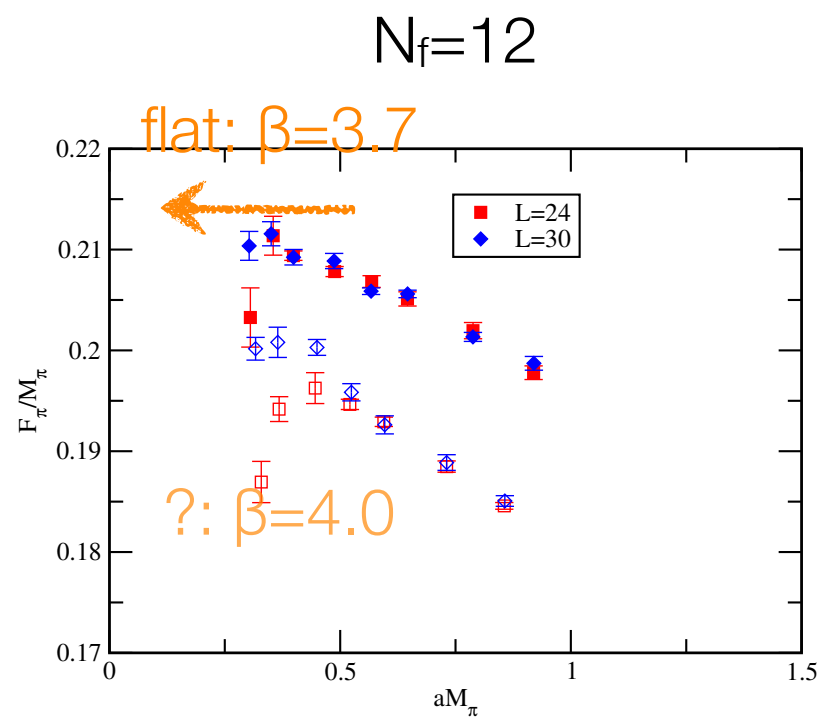
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$$\Rightarrow a(\beta=3.7) / a(\beta=4.0) > 1$$

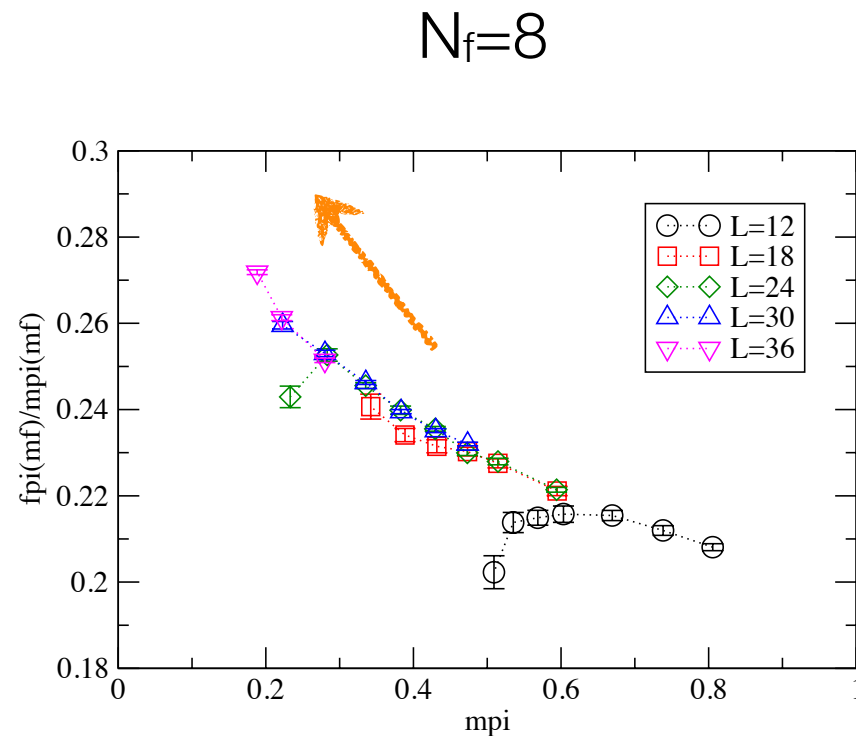
- consistent with UV asymptotic freedom

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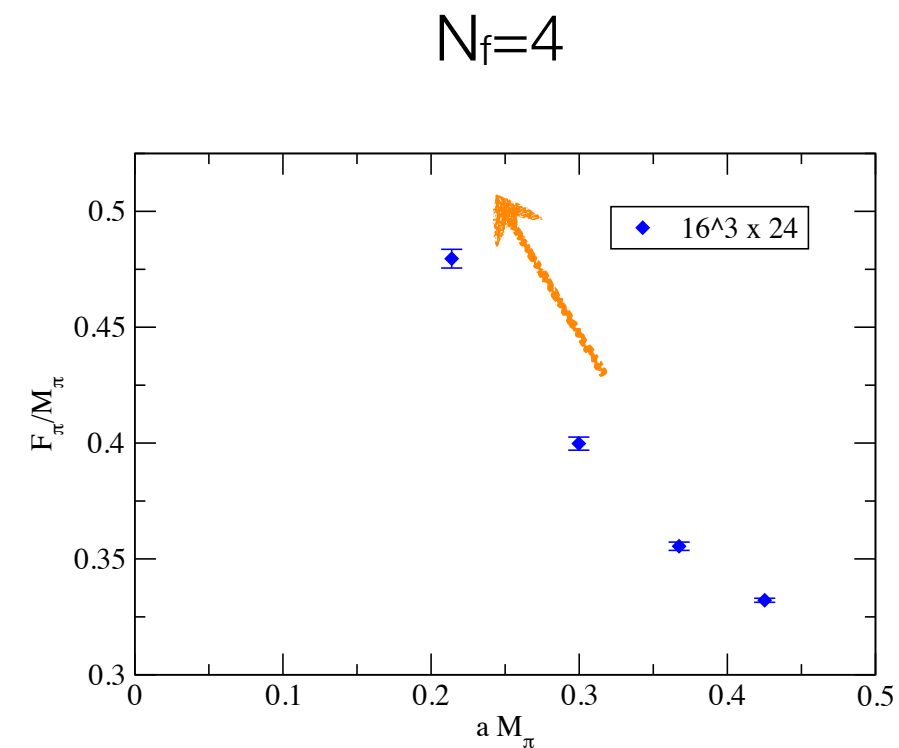
a crude analysis: F_π/M_π vs M_π leads to a likely scenario



- conformality



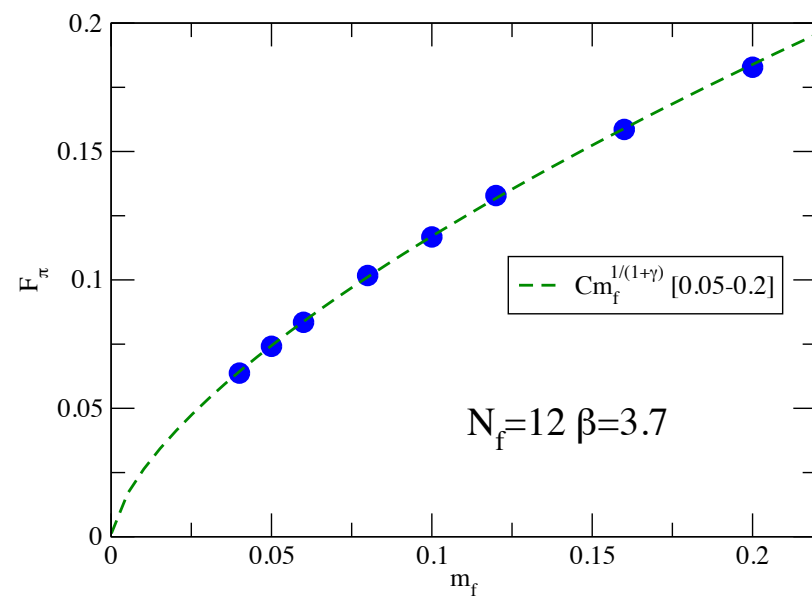
- ~~chiral symmetry~~



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F_π vs m_f

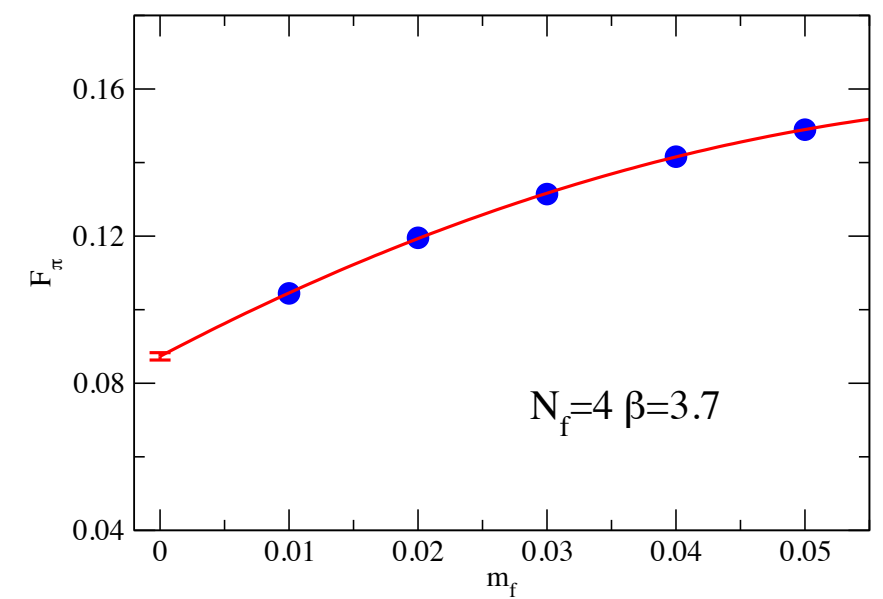
$N_f=12$



- conformality
- $F_\pi \rightarrow C m_f^{1/(1+\gamma)}$

- $\gamma \sim 0.5$

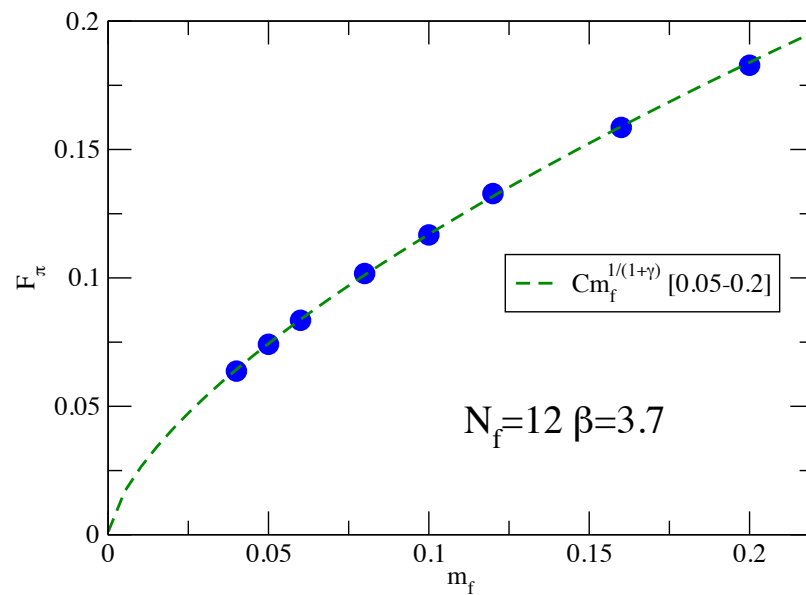
$N_f=4$



- ~~chiral symmetry~~
- $F_\pi \rightarrow F \neq 0$

F_π vs m_f

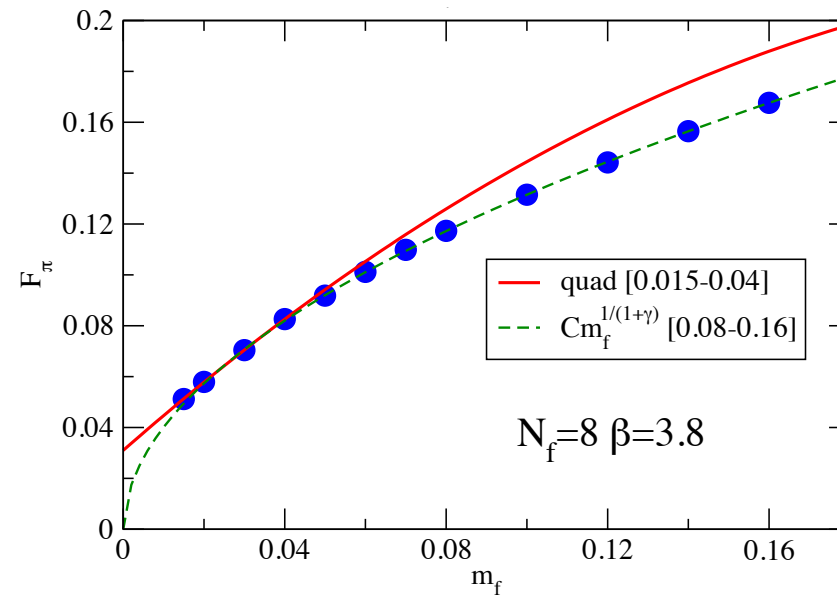
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- conformality
- $F_\pi \rightarrow C m_f^{1/(1+\gamma)}$

- $\gamma \sim 0.5$

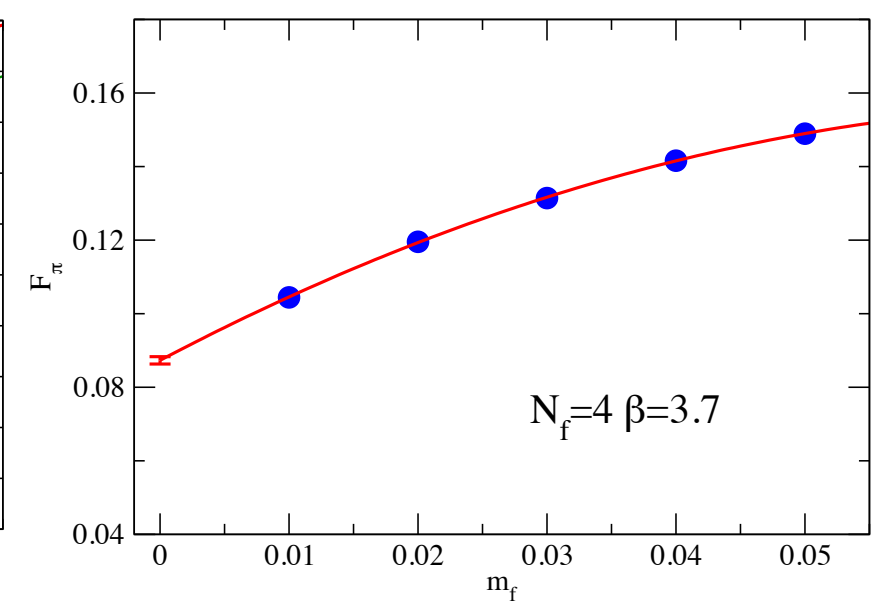
$N_f=8$



- ~~chiral symmetry~~
- $F_\pi \rightarrow F \neq 0 \quad m_f \rightarrow 0$
- $F_\pi \rightarrow C m_f^{1/(1+\gamma)}$
intermediate m_f

- $\gamma \sim 0.9$

$N_f=4$



- ~~chiral symmetry~~
- $F_\pi \rightarrow F \neq 0$

conformal (finite size) scaling

- Scaling dimension at IR fixed point [Wilson-Fisher]; Hyper Scaling [Miransky]
- mass dependence is described by anomalous dimensions at IRFP

- quark mass anomalous dimension γ^*

- operator anomalous dimension

- hadron mass and pion decay constant obey same scaling

$$M_H \propto m_f^{\frac{1}{1+\gamma^*}}$$

$$F_\pi \propto m_f^{\frac{1}{1+\gamma^*}}$$

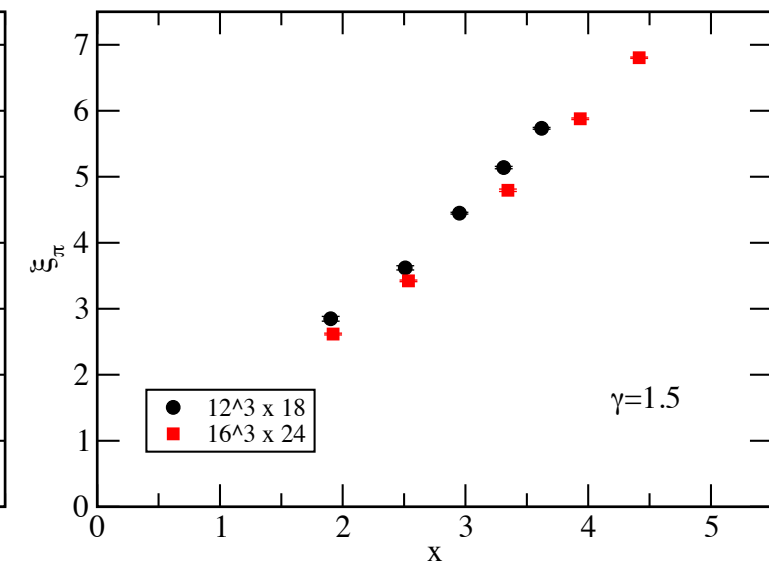
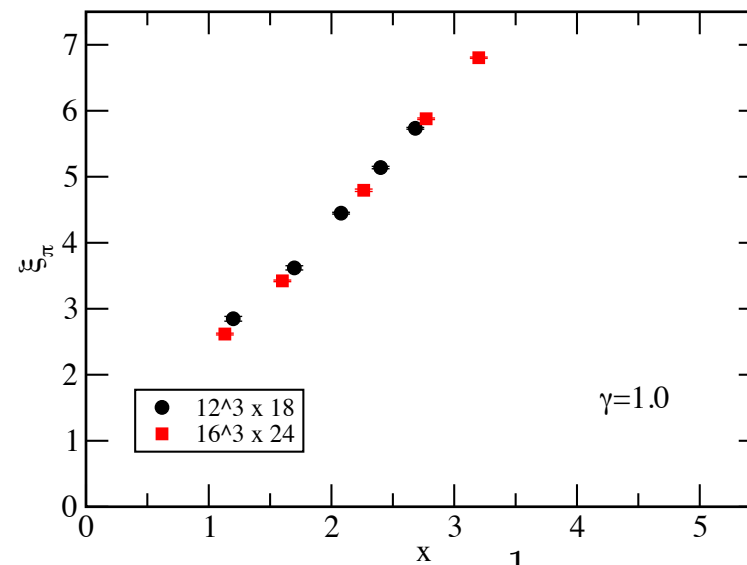
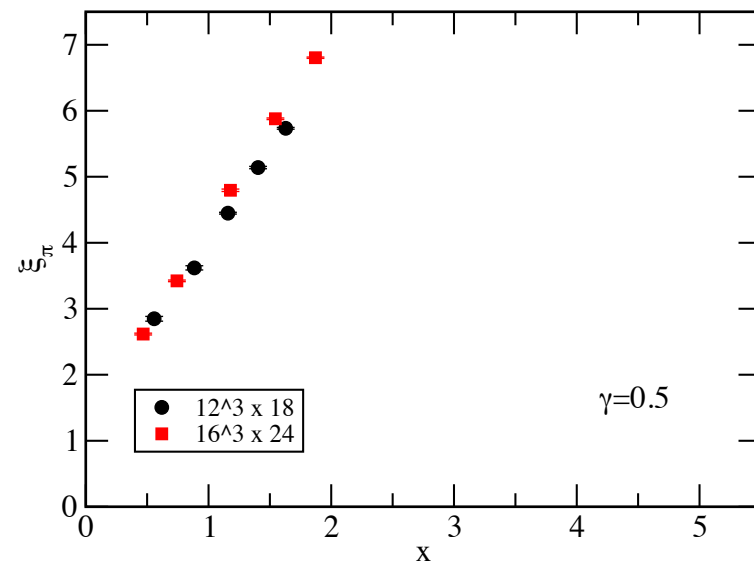
- **finite size scaling** in a L^4 box (DeGrand; Zwicky; Del Debbio et al)

- scaling variable: $x = L m_f^{\frac{1}{1+\gamma^*}}$

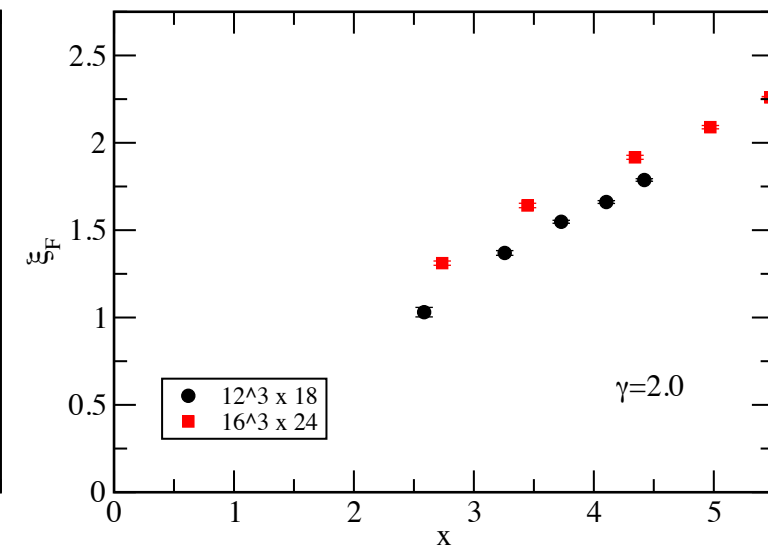
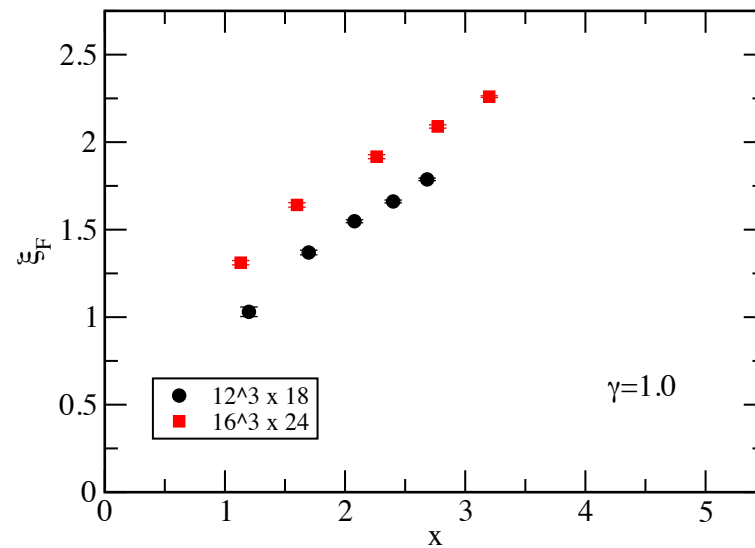
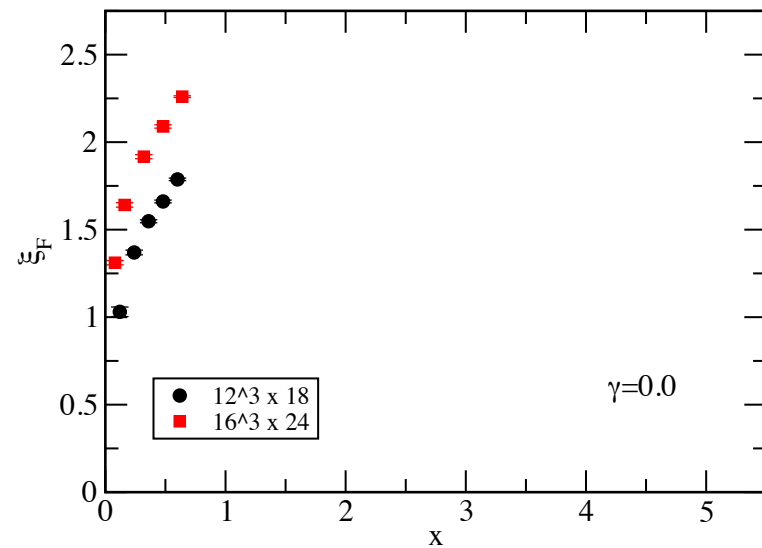
$$L \cdot M_H = f_H(x)$$

$$L \cdot F_\pi = f_F(x)$$

$N_f=4$ see if data align at some γ

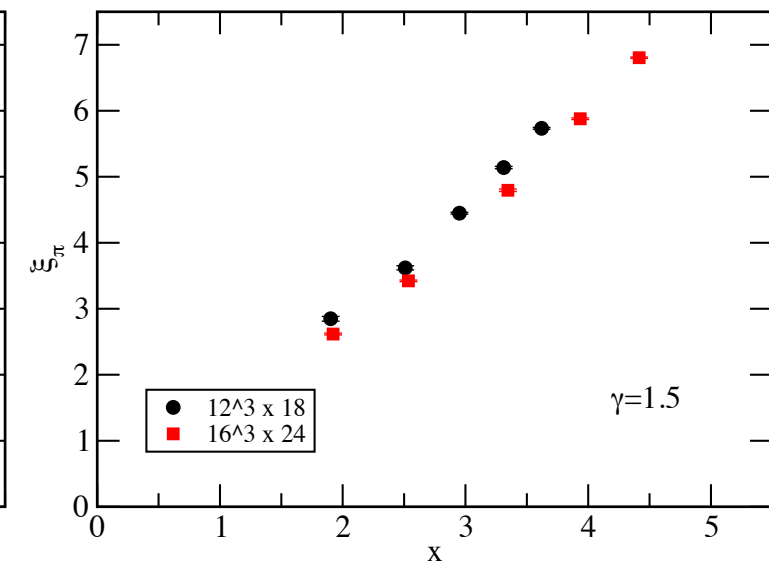
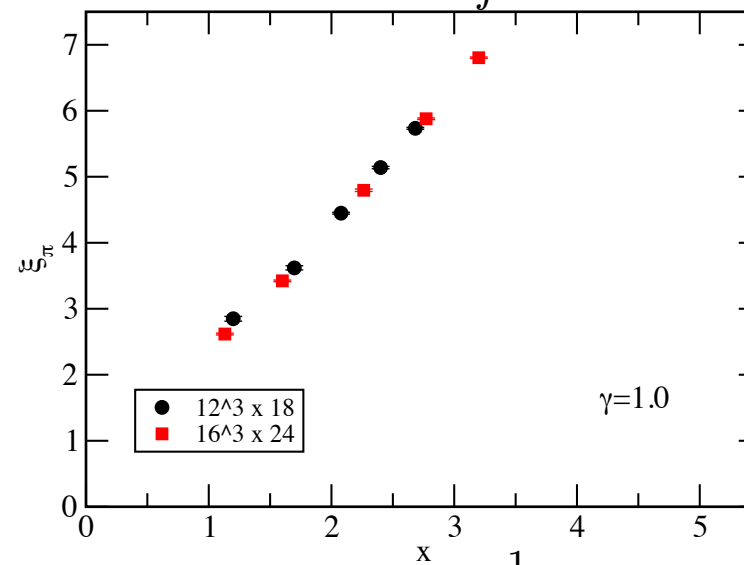
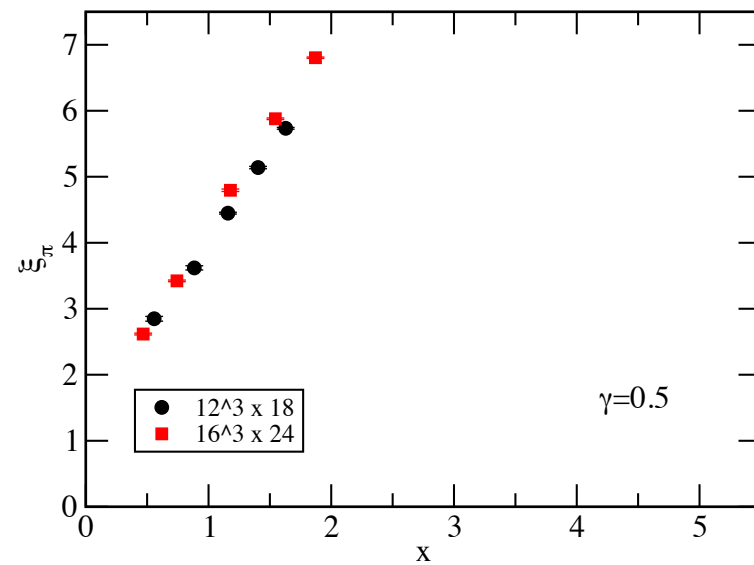


$$x = Lm_f^{\frac{1}{1+\gamma}}$$

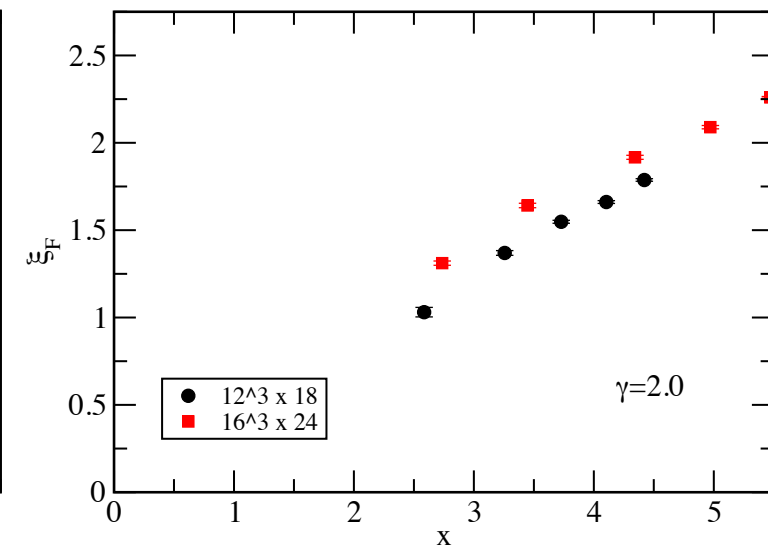
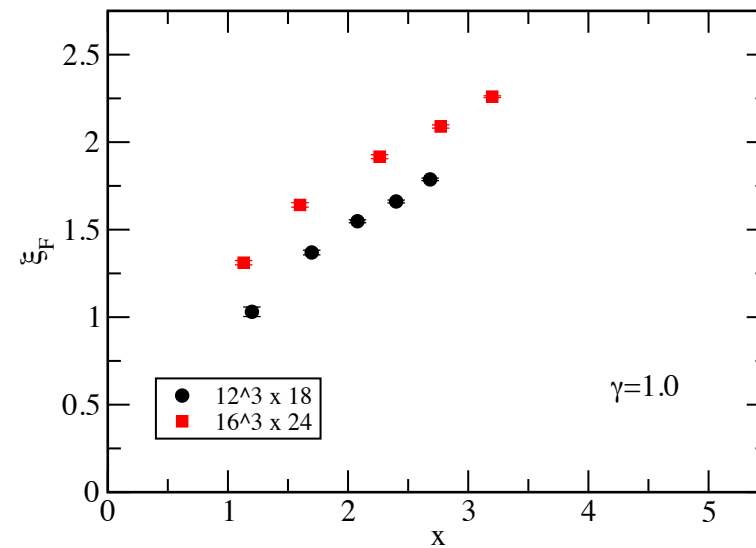
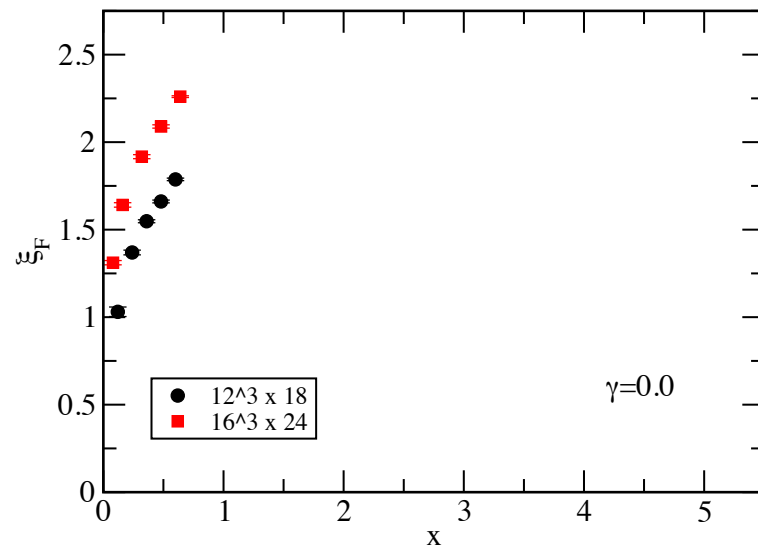


$N_f=4$ see if data align at some γ

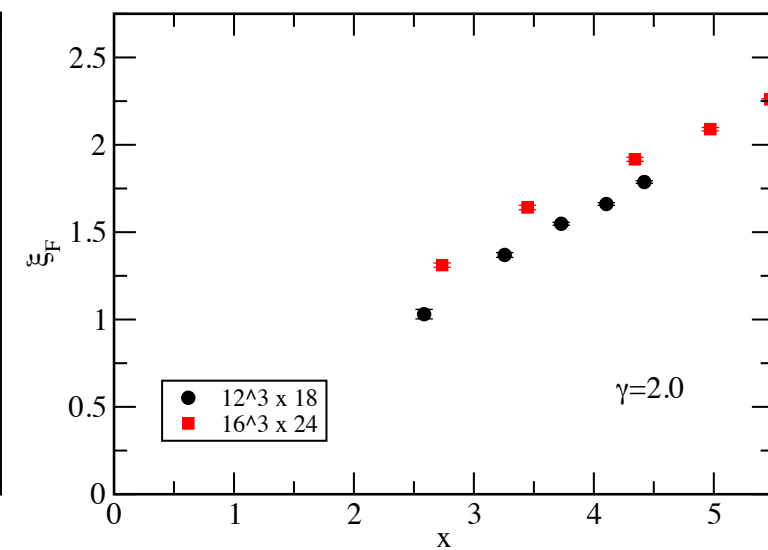
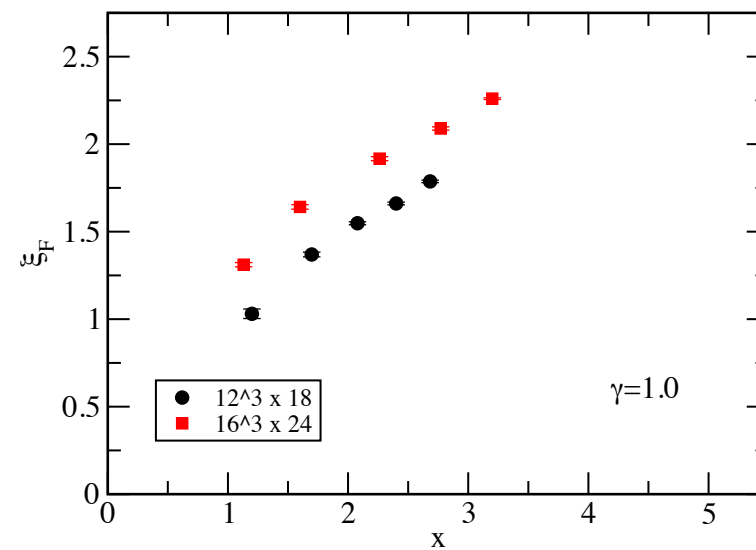
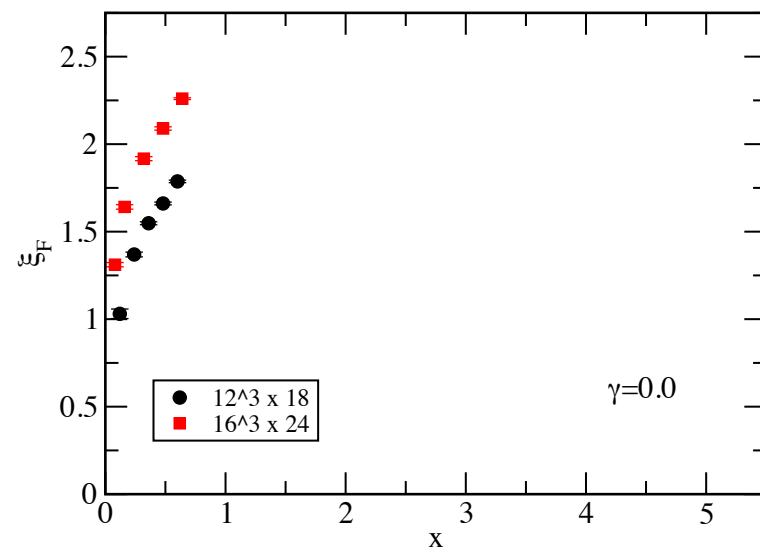
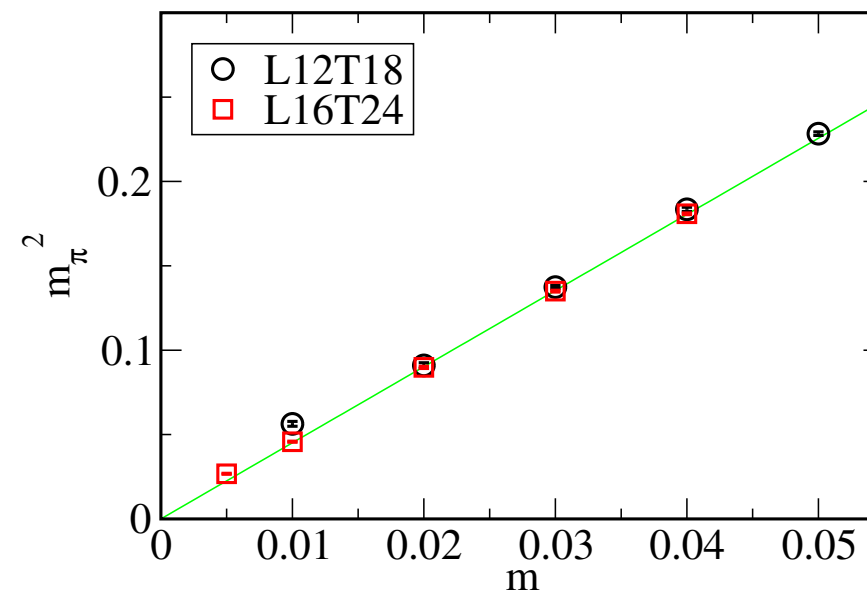
$$M_\pi L \propto m_f^{1/2} L$$



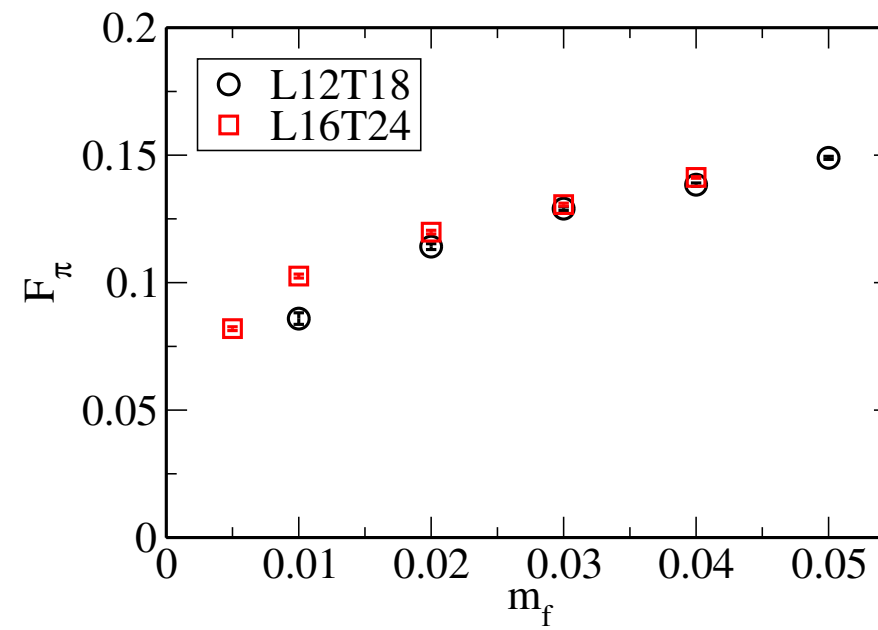
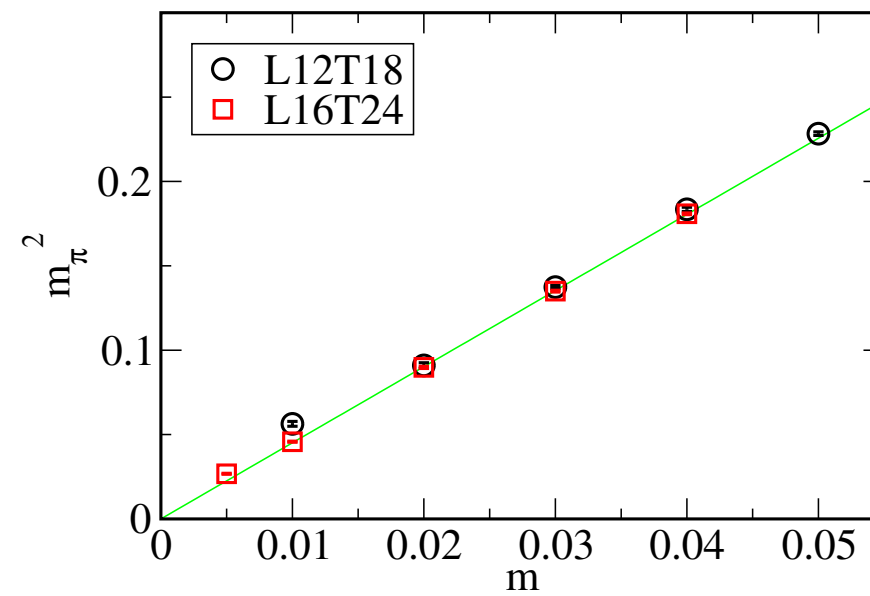
$$x = L m_f^{\frac{1}{1+\gamma}}$$



$N_f=4$ see if data align at some γ

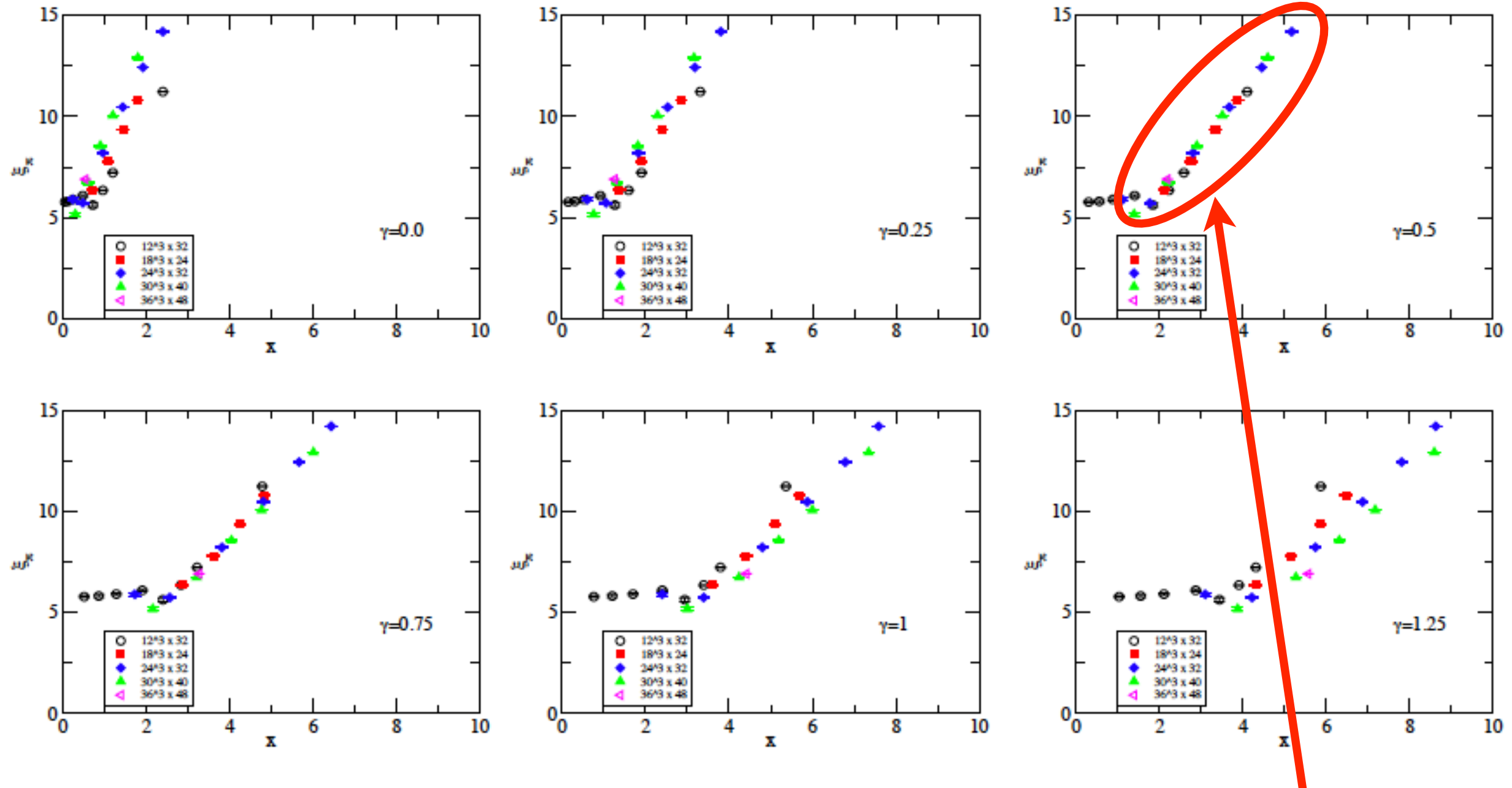


$N_f=4$ see if data align at some γ



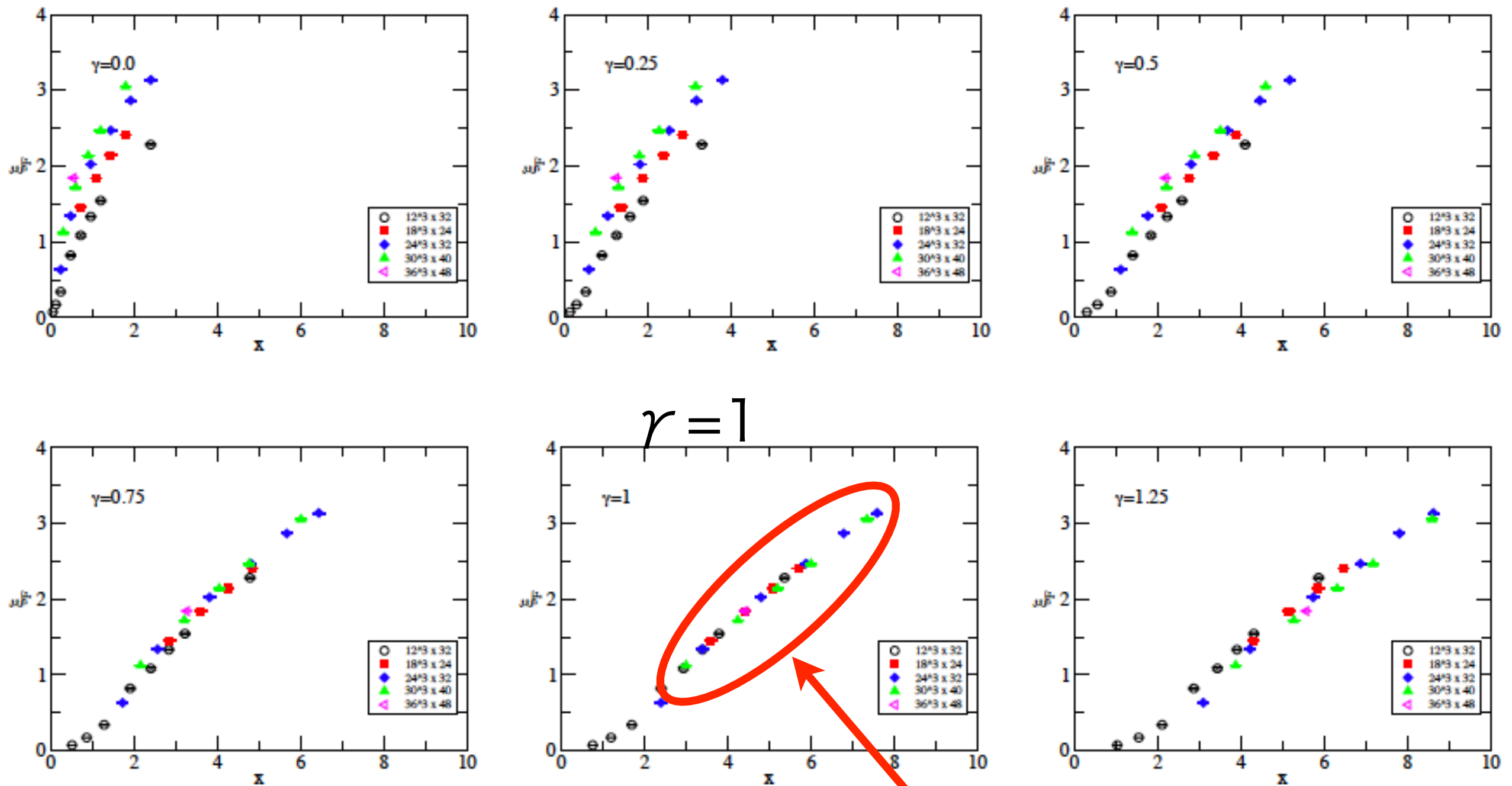
$N_f=8$ see if data align at some γ : M_π

$\gamma=0.5$



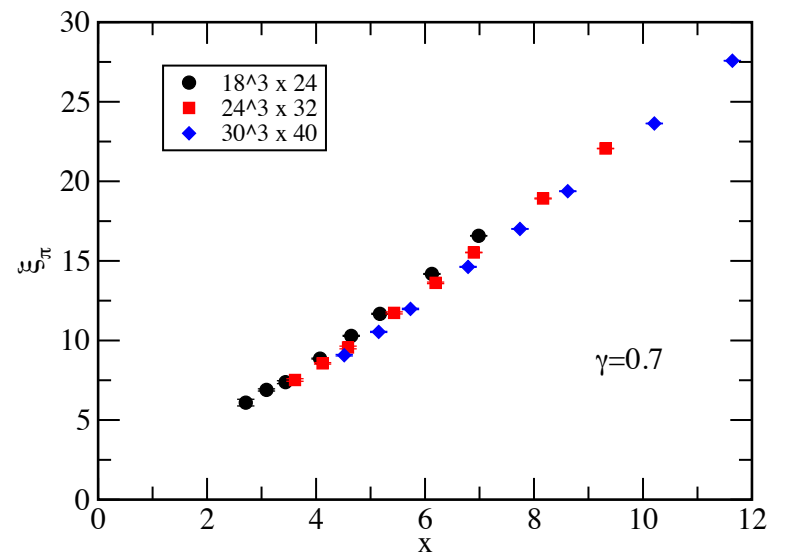
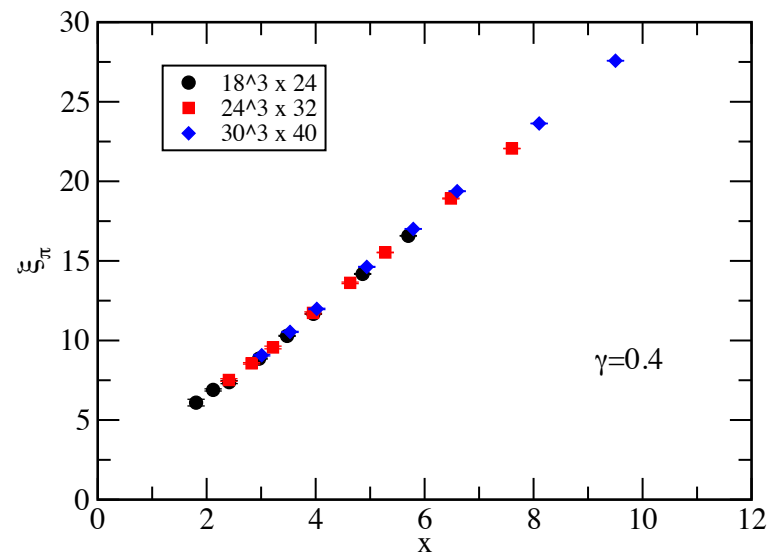
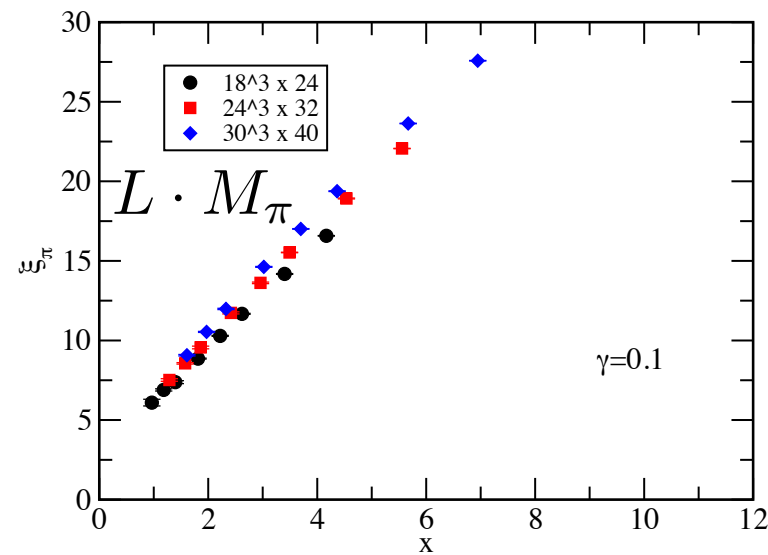
good alignment

$N_f=8$ see if data align at some γ : F_π

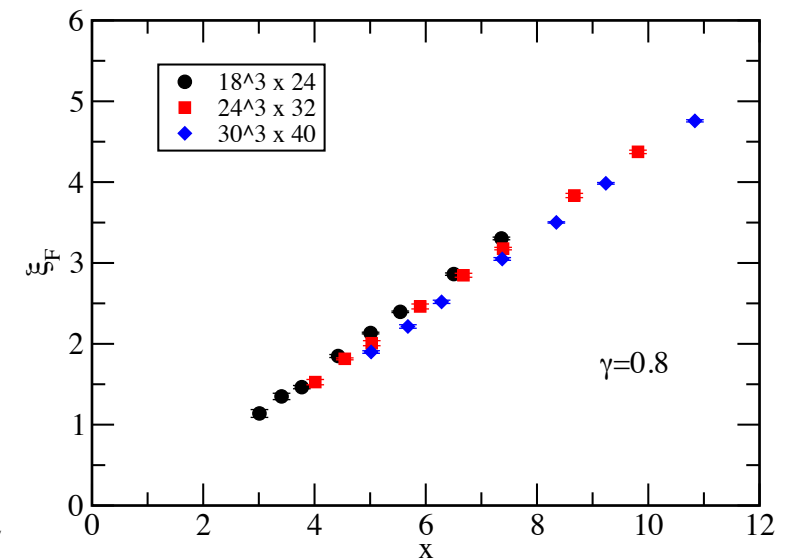
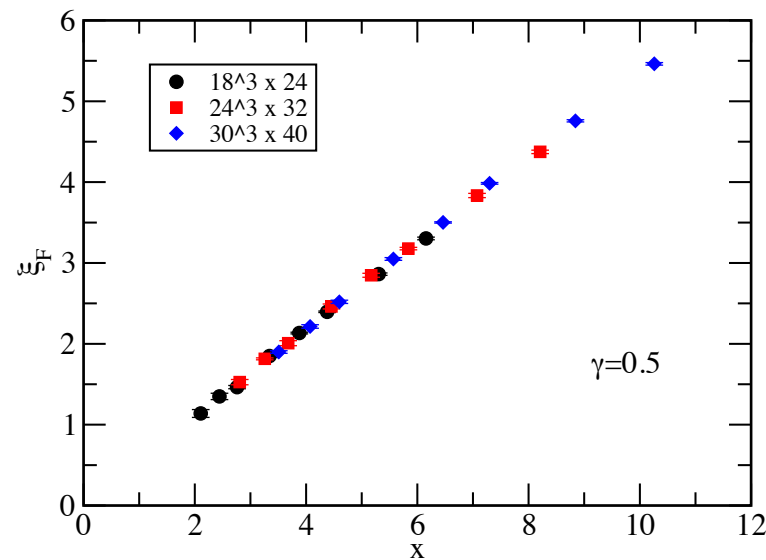
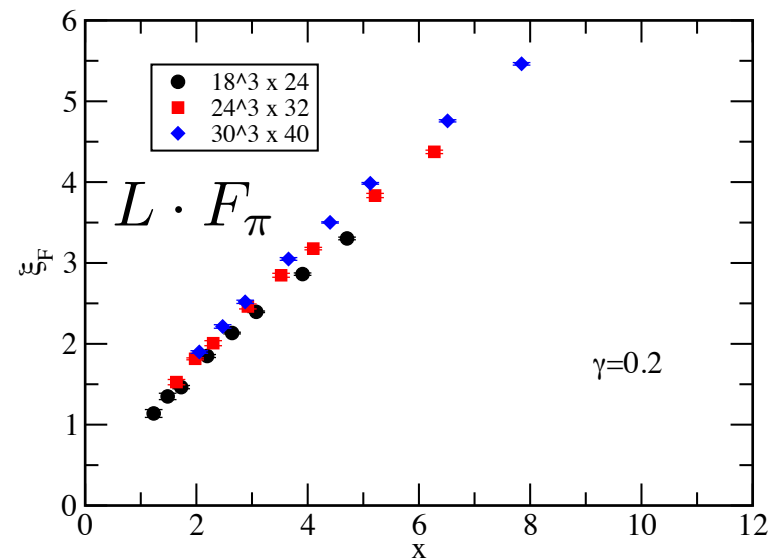


good alignment

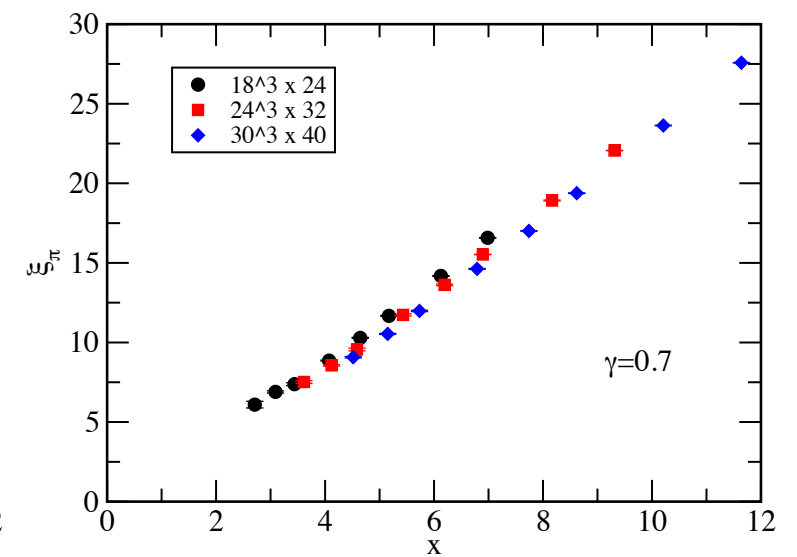
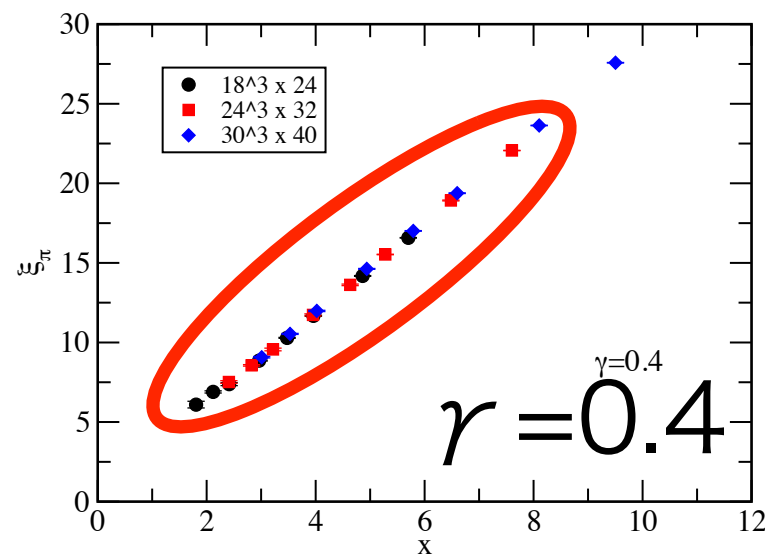
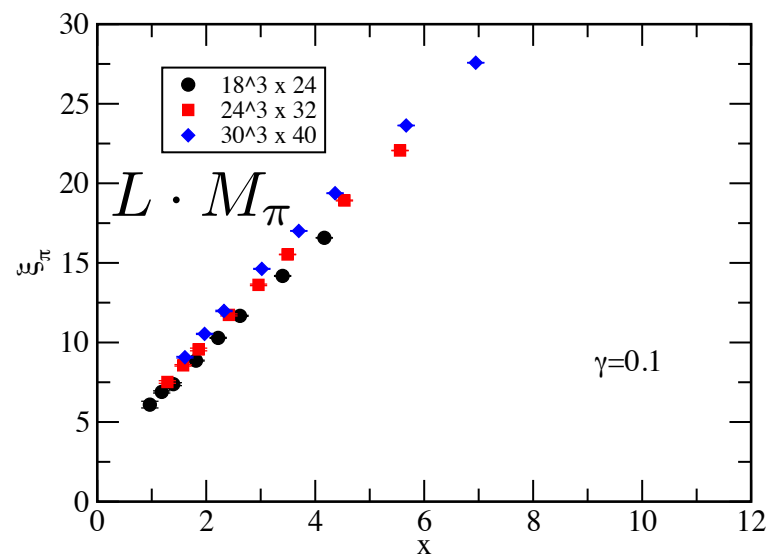
$N_f=12$ see if data align at some γ



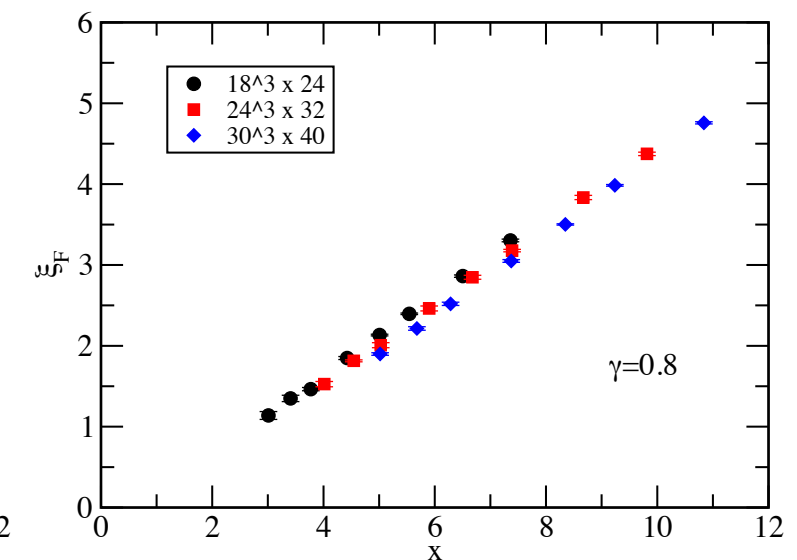
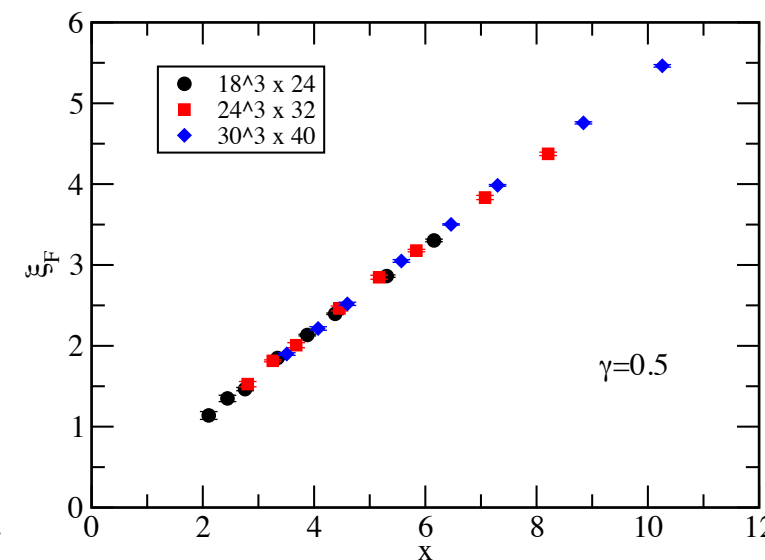
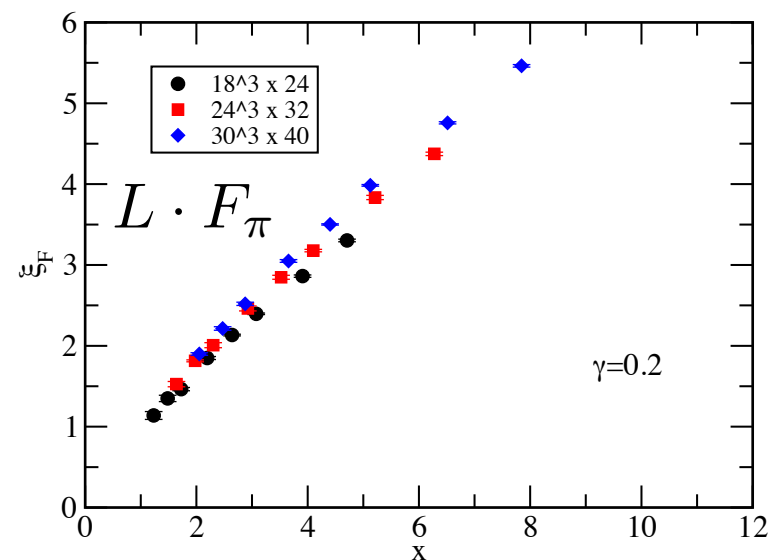
$$x = L m_f^{\frac{1}{1+\gamma}}$$



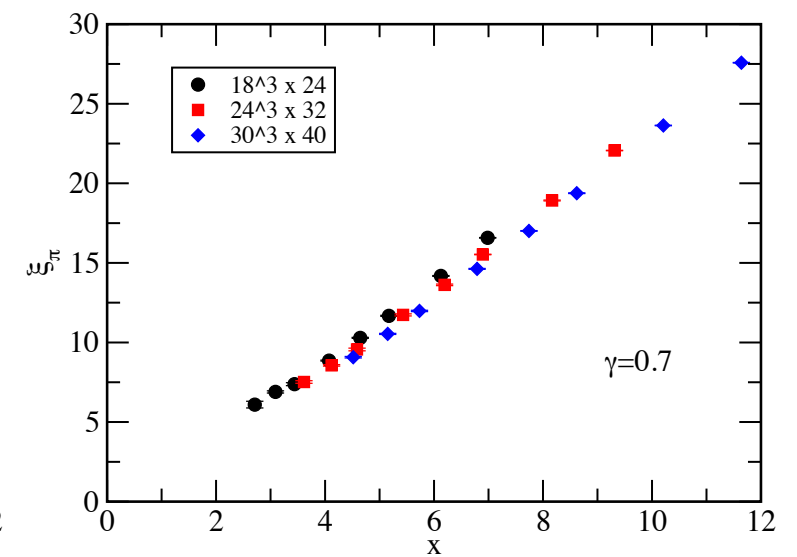
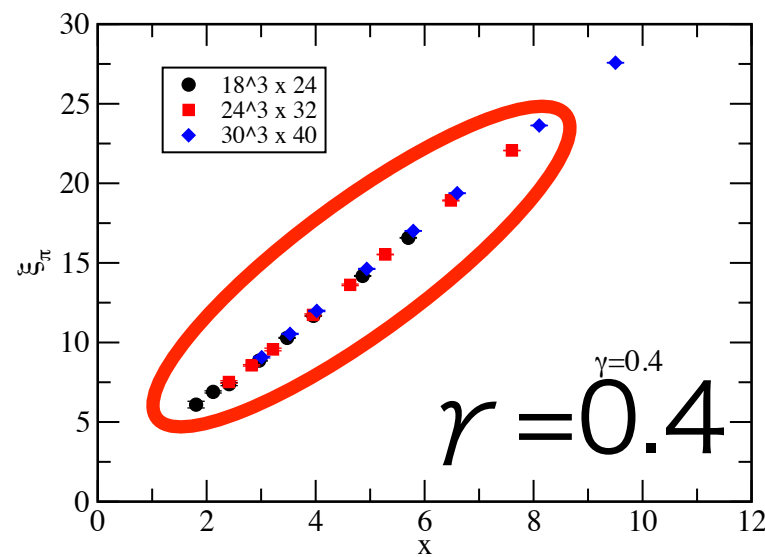
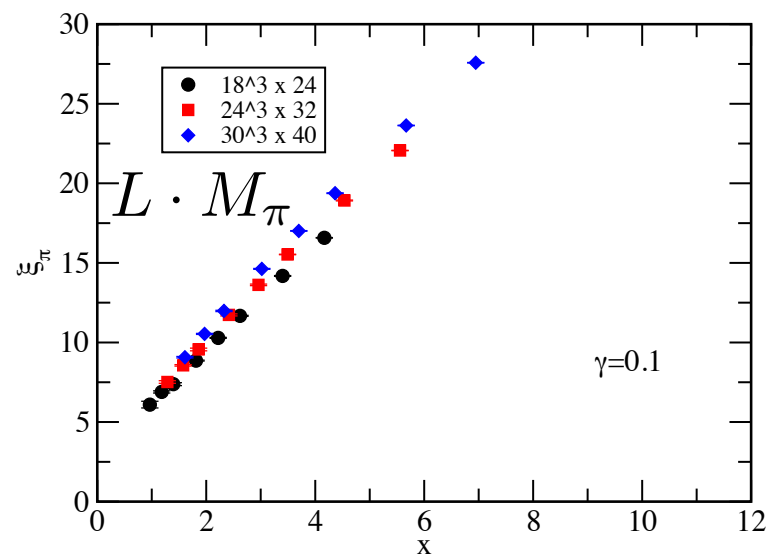
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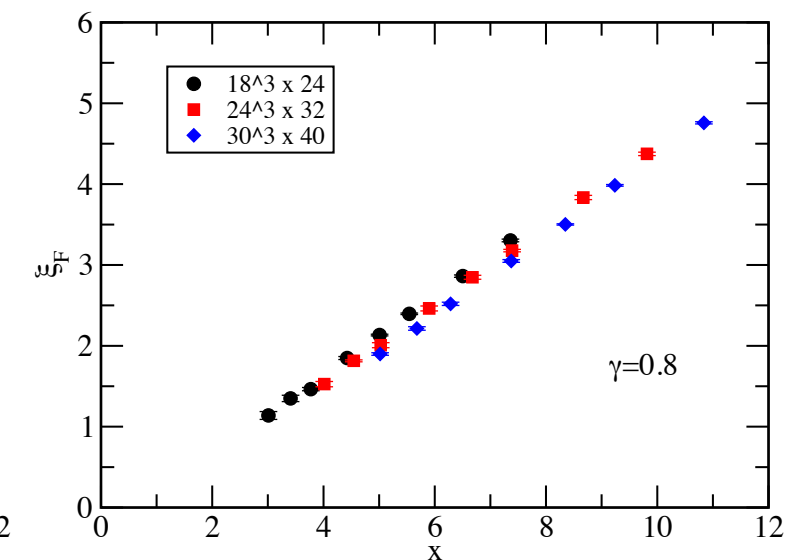
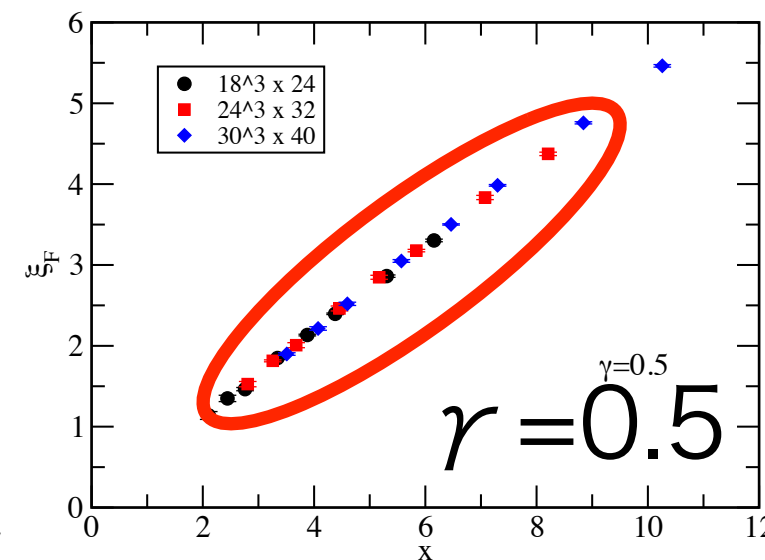
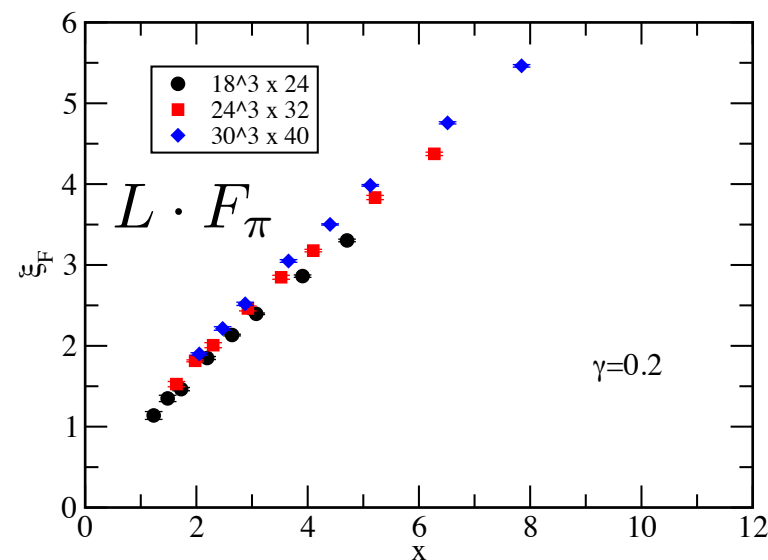
$$x = L m_f^{\frac{1}{1+\gamma}}$$



$N_f=12$ see if data align at some γ



$$x = L m_f^{\frac{1}{1+\gamma}}$$



measure of the “alignment”
without resorting to a model

measure of the “alignment” without resorting to a model

- γ of optimal alignment will minimize:

$$P_p(\gamma) = \frac{1}{\mathcal{N}} \sum_K \sum_{j \notin K} \frac{|\xi_p^j - f_p^{(K)}(x_j)|^2}{\delta^2 \xi_p^j}$$

- $\xi_p = LM_p$ for $p = \pi, \rho$; $\xi_F = LF_\pi$
- $f_p(x)$: interpolation linear
 - (quadratic for a systematic error)
- if ξ^j is away from $f(x_i)$ by $\delta \xi^j$ as average $\rightarrow P=1$
- optimal γ from the minimum of P
- similar definition of the measure: DeGrand, Giedt & Weinberg

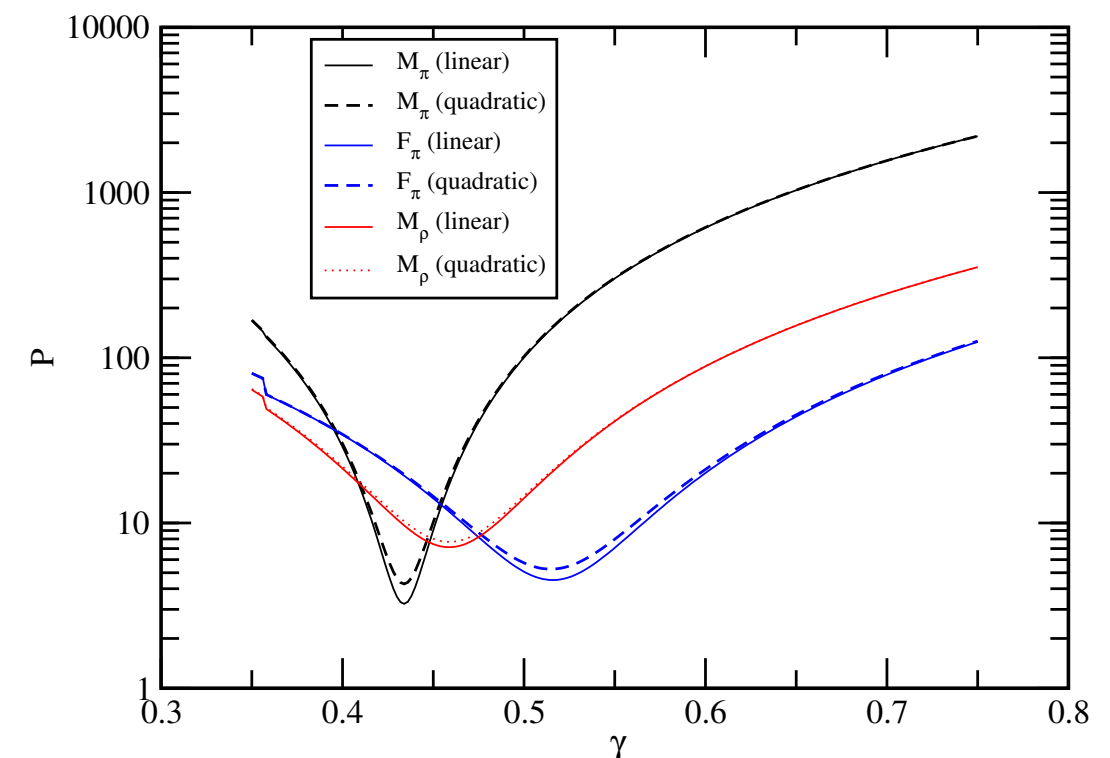
measure of the “alignment” without resorting to a model

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$P(\gamma)$ for M_π, F_π, M_ρ at $\beta=3.7$



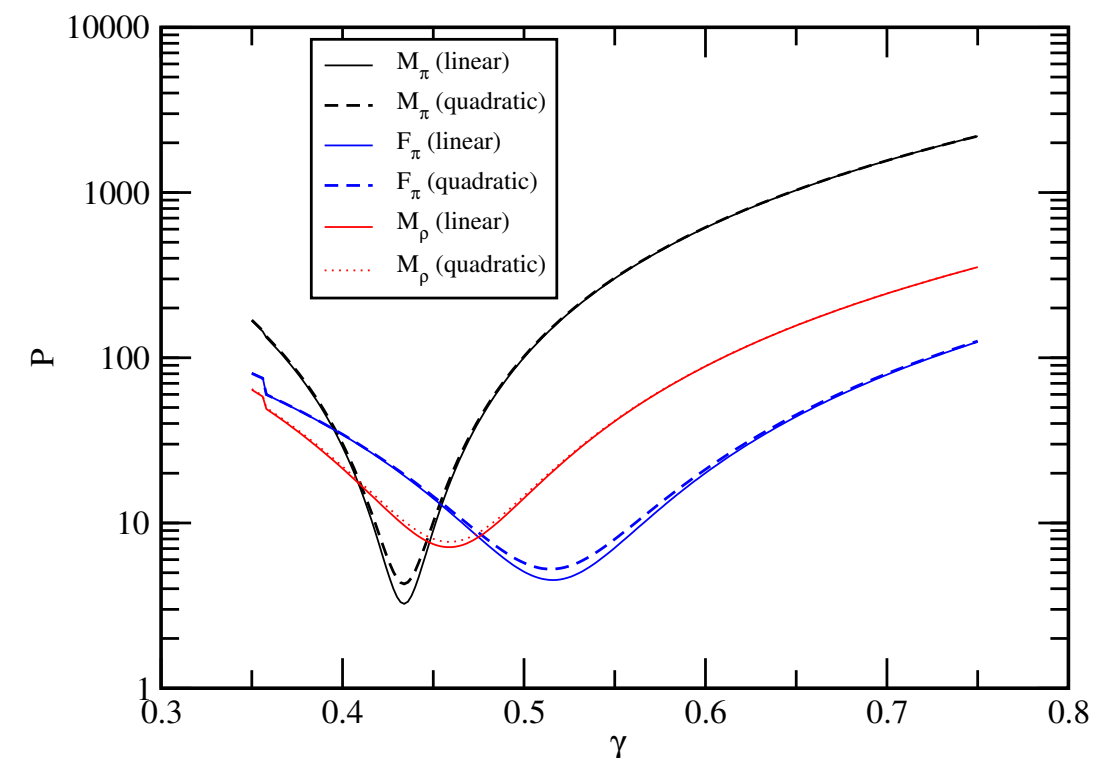
measure of the “alignment” without resorting to a model

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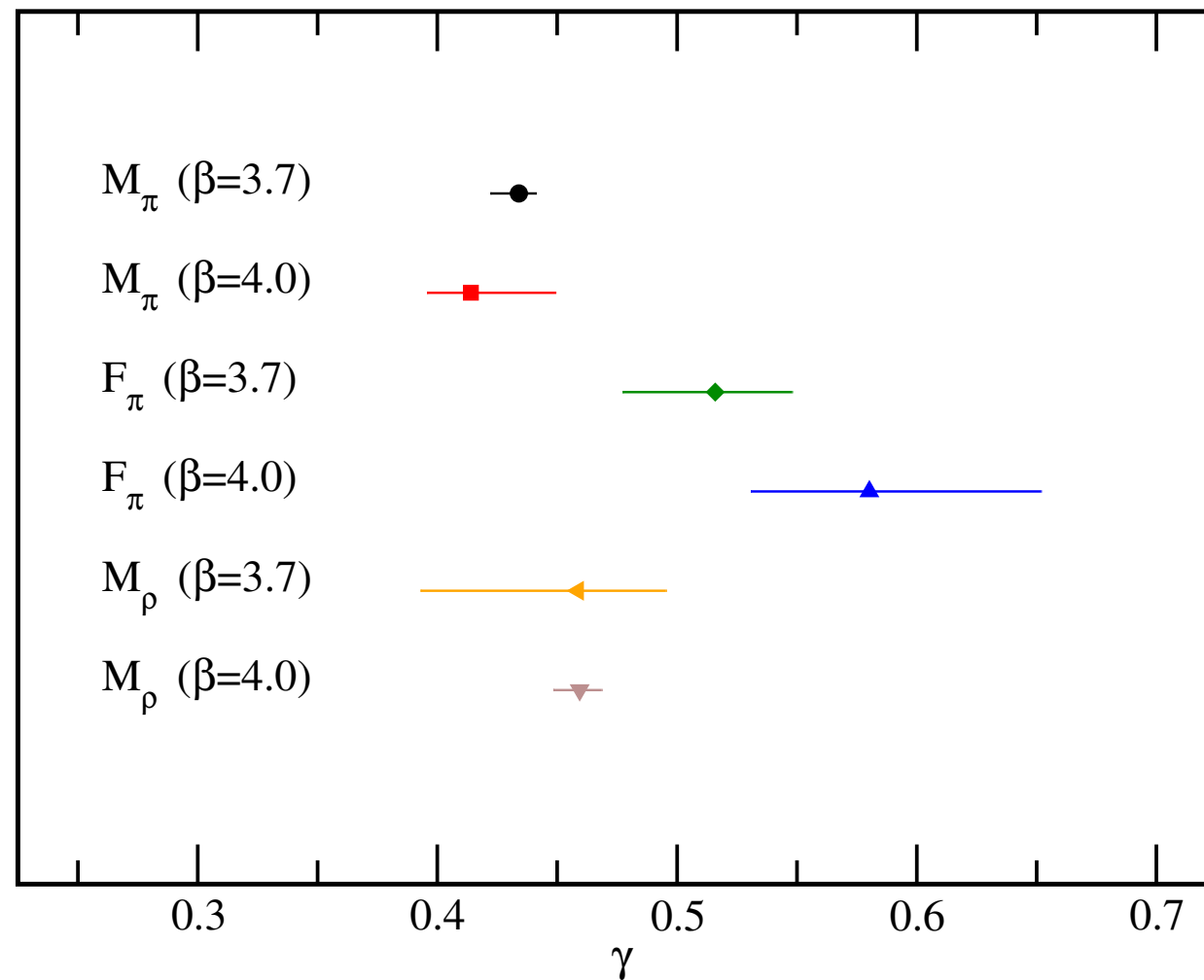
$$P_p(\gamma) = \frac{1}{\mathcal{N}} \sum_K \sum_{j \notin K} \frac{|\xi_p^j - f_p^{(K)}(x_j)|^2}{\delta^2 \xi_p^j}$$

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- optimal γ from the minimum of P
- similar definition of the measure: DeGrand, Giedt & Weinberg
- systematic error due to small L , large m estimated by examining the x and L range dependence

$P(\gamma)$ for M_π, F_π, M_ρ at $\beta=3.7$

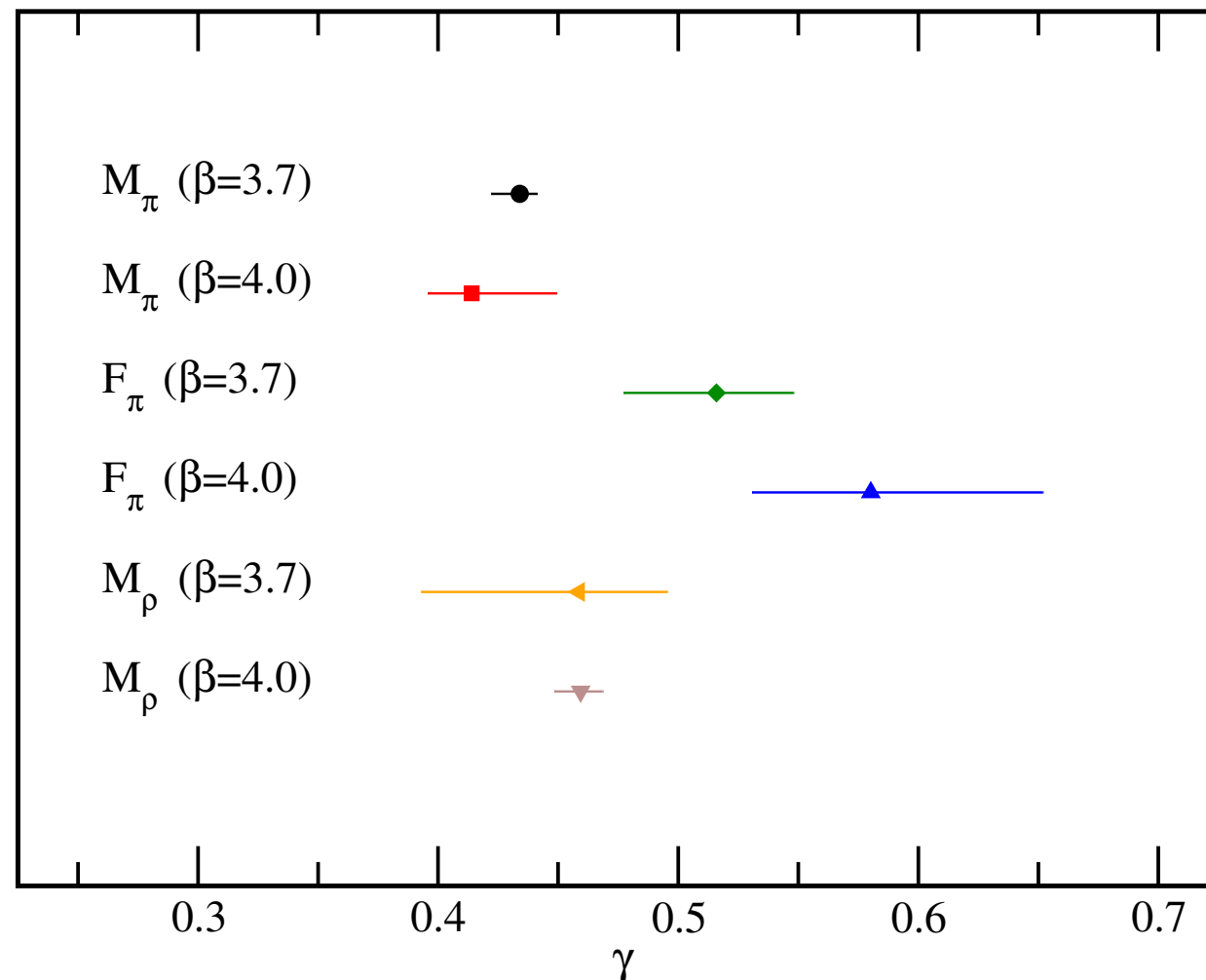


summary of γ from $P(\gamma)$ for $N_f=12$



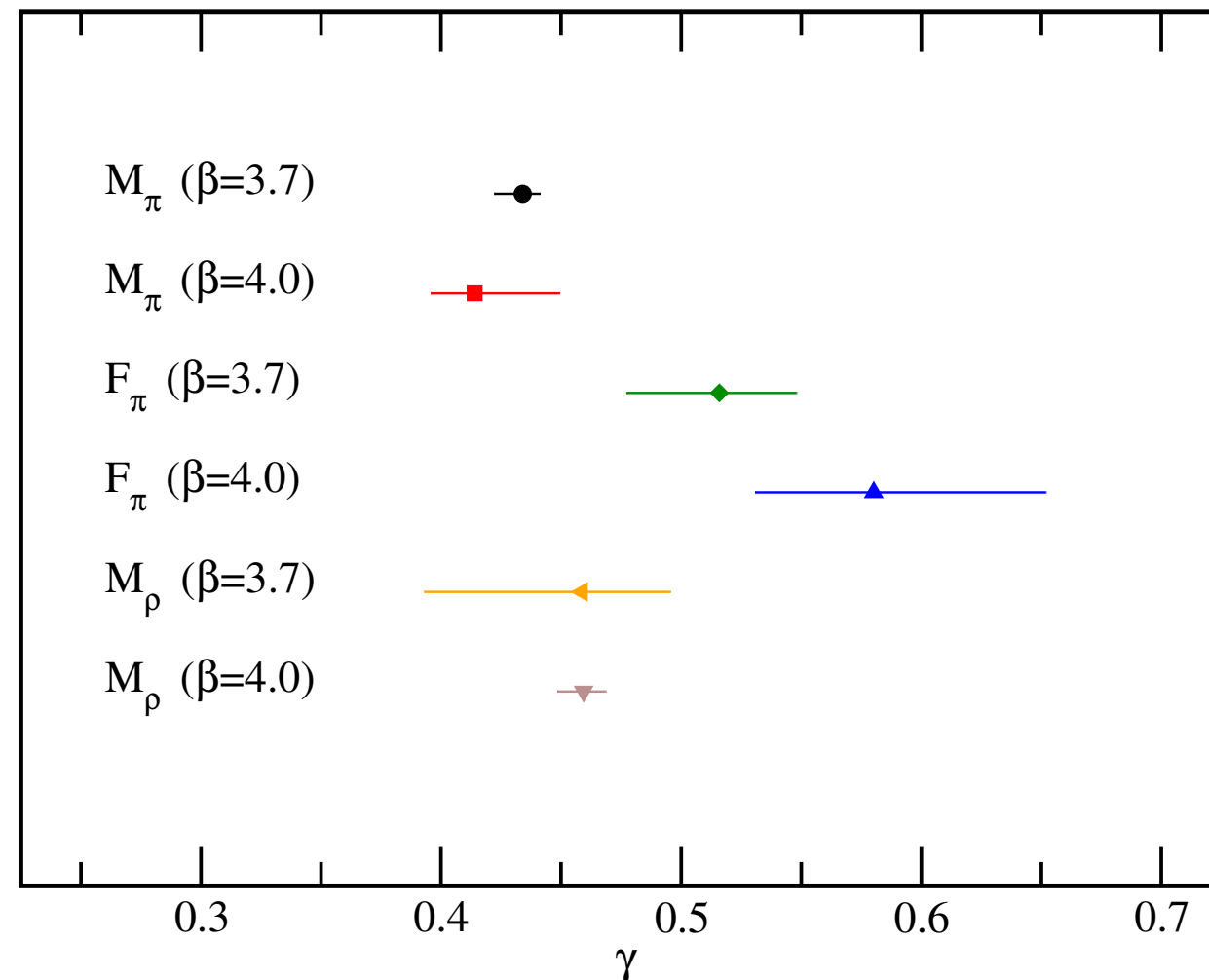
PRD 86 (2012)

summary of γ from $P(\gamma)$ for $N_f=12$



- γ : consistent with 2 σ level except for F_π at $\beta=4.0$

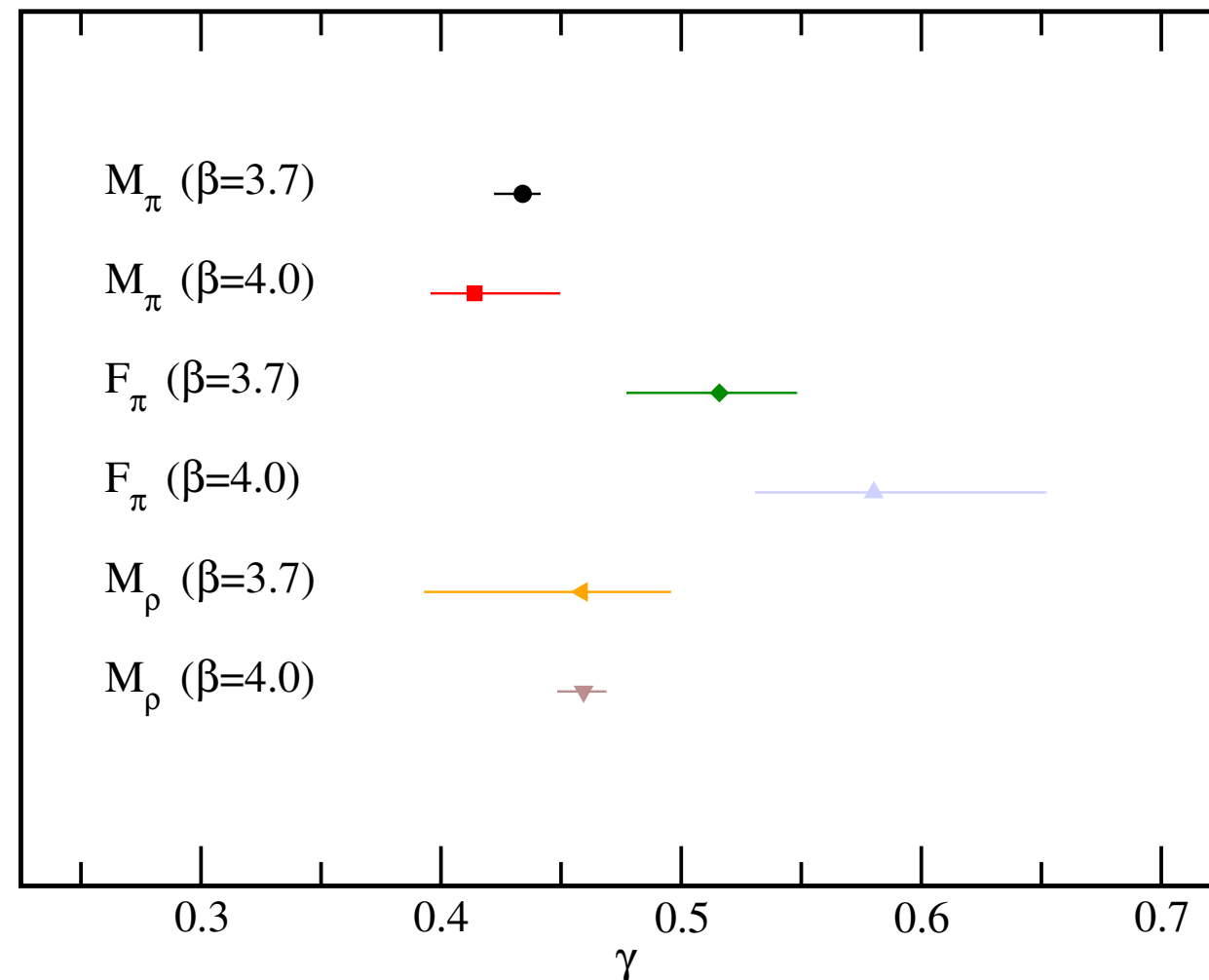
summary of γ from $P(\gamma)$ for $N_f=12$



PRD 86 (2012)

- γ : consistent with 2 σ level except for F_π at $\beta=4.0$
- F_π at $\beta=4.0$ speculated to be out of the scaling region

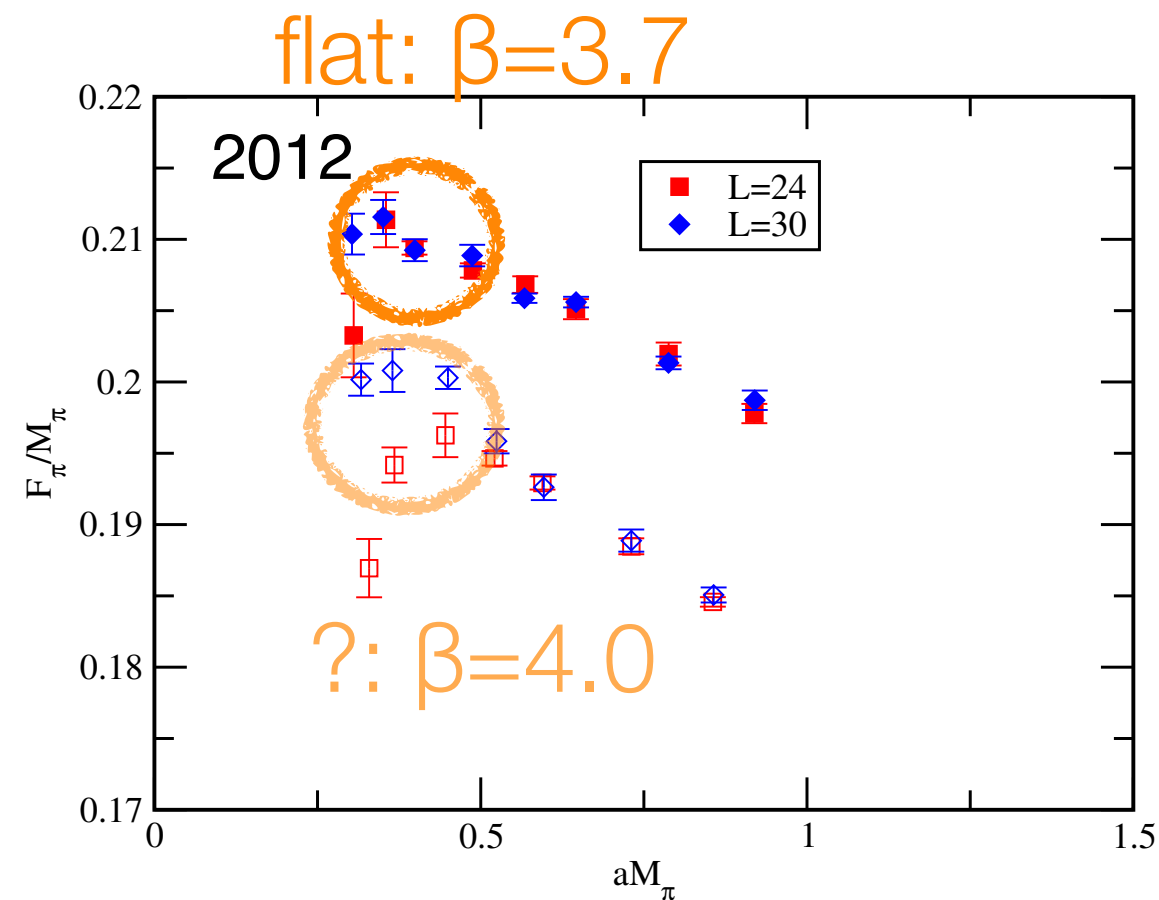
summary of γ from $P(\gamma)$ for $N_f=12$



PRD 86 (2012)

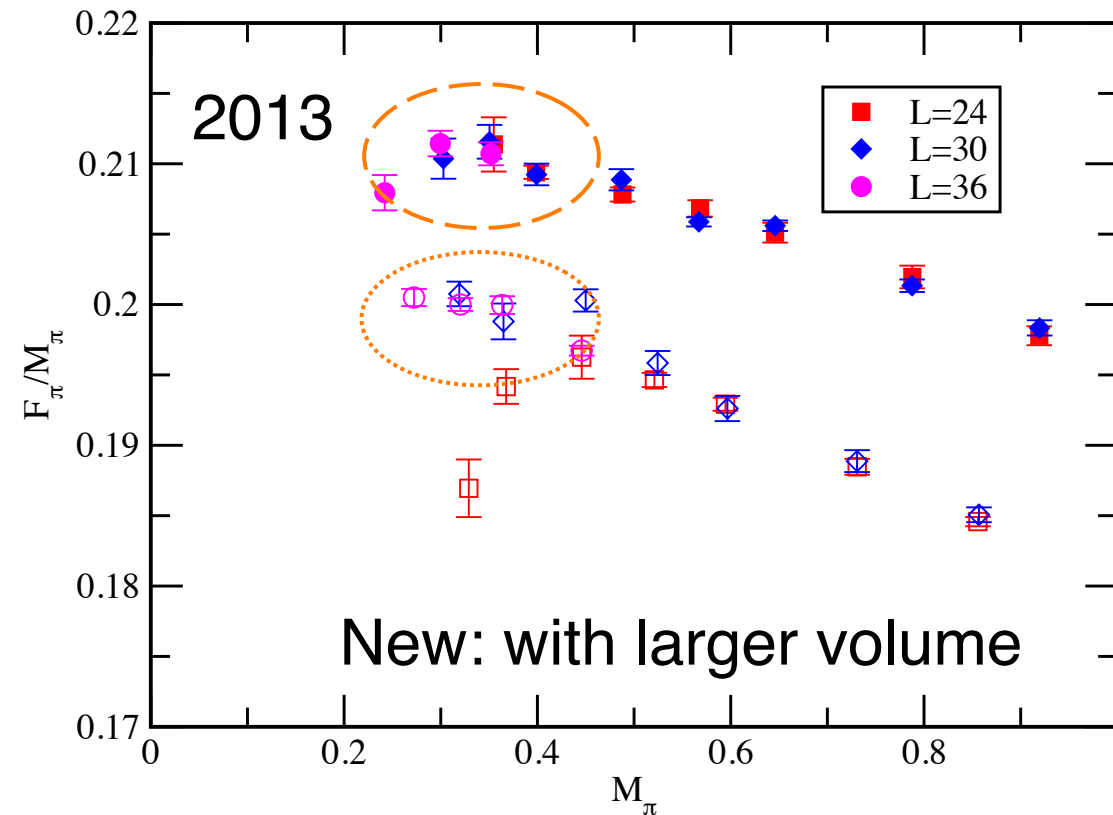
- γ : consistent with 2 σ level except for F_π at $\beta=4.0$
- F_π at $\beta=4.0$ speculated to be out of the scaling region
- universal low energy behavior: good with $0.4 < \gamma^* < 0.5$

summary of γ from $P(\gamma)$ for $N_f=12$



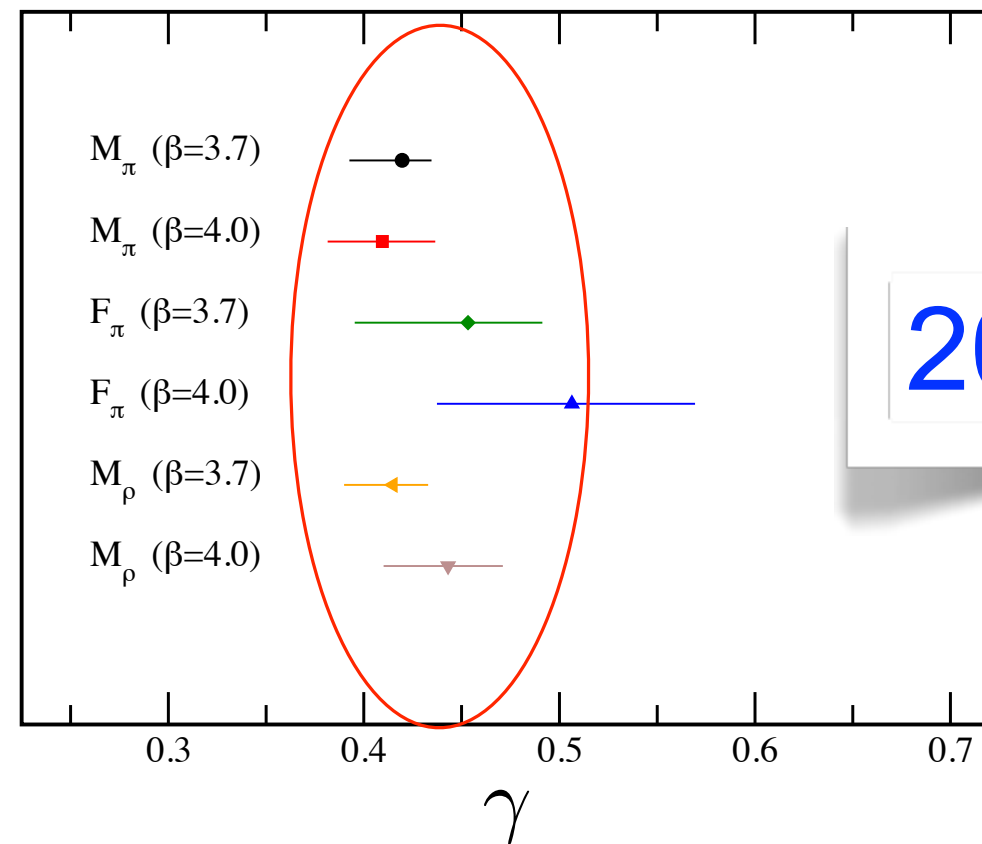
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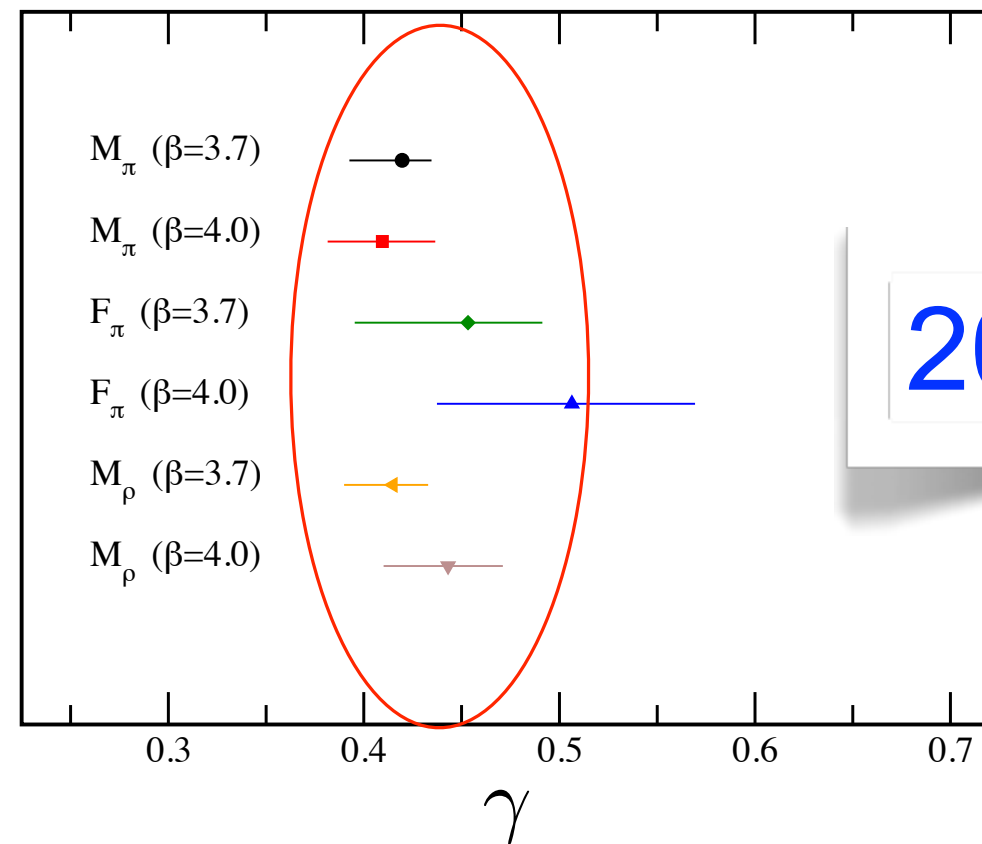
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2013 Update

- γ : consistent with 2 σ level except for F_π at $\beta=4.0$
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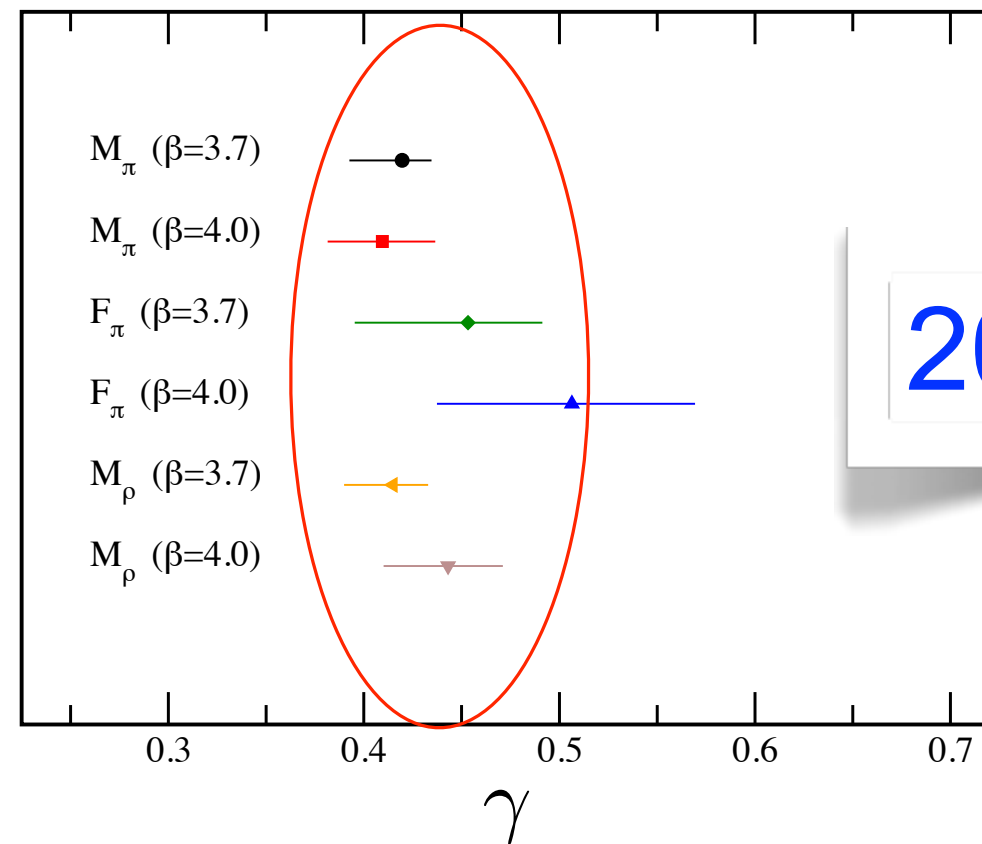
summary of γ from $P(\gamma)$ for $N_f=12$



2013 Update

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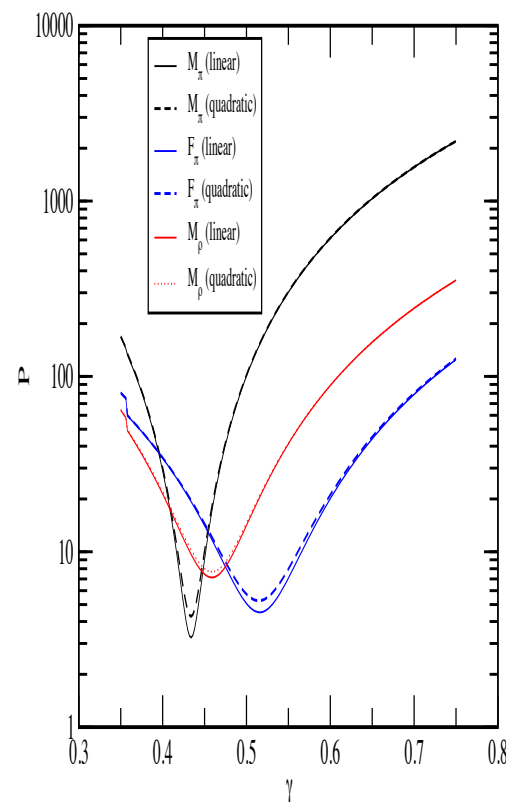
summary of γ from $P(\gamma)$ for $N_f=12$



2013 Update

- γ : consistent with 2 σ level except for F_π at $\beta=4.0$
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$P(\gamma)$ analysis for $N_f=8$

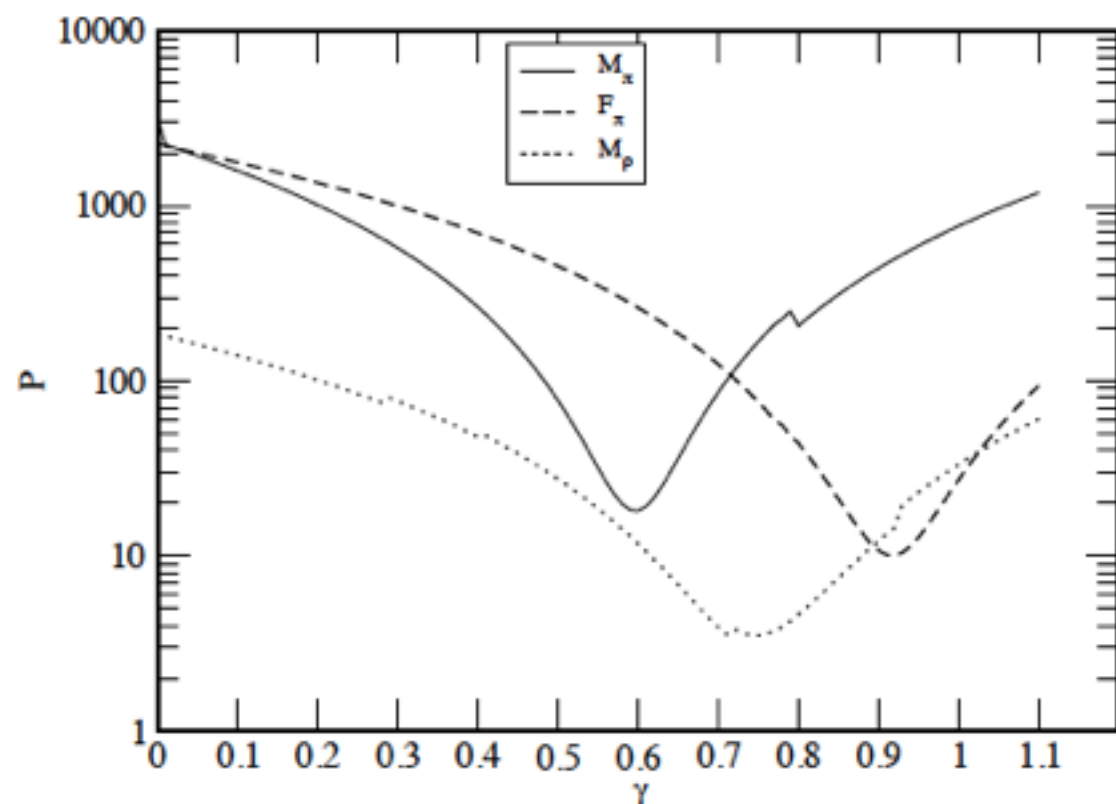


$N_f=12$

quantity	γ
M_π	0.434(4)
F_π	0.516(12)
M_ρ	0.459(8)

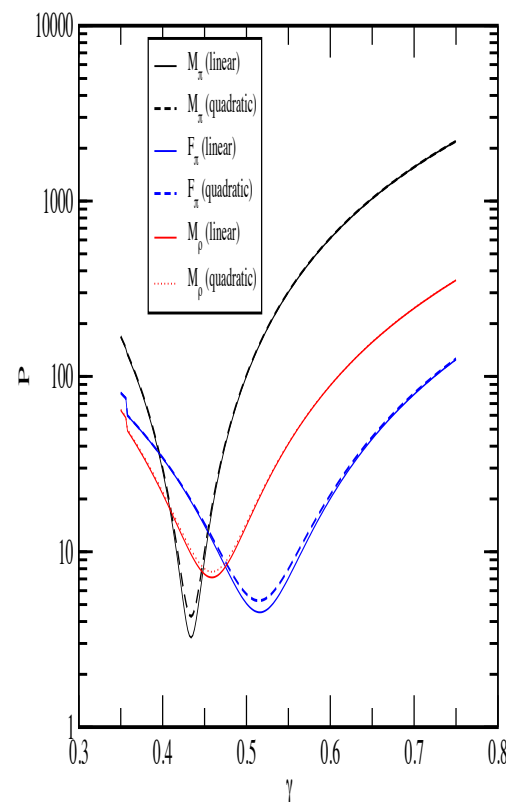
statistical error only

$N_f=8$



quantity	γ
M_π	0.593(2)
F_π	0.955(4)
M_ρ	0.820(20)

$P(\gamma)$ analysis for $N_f=8$



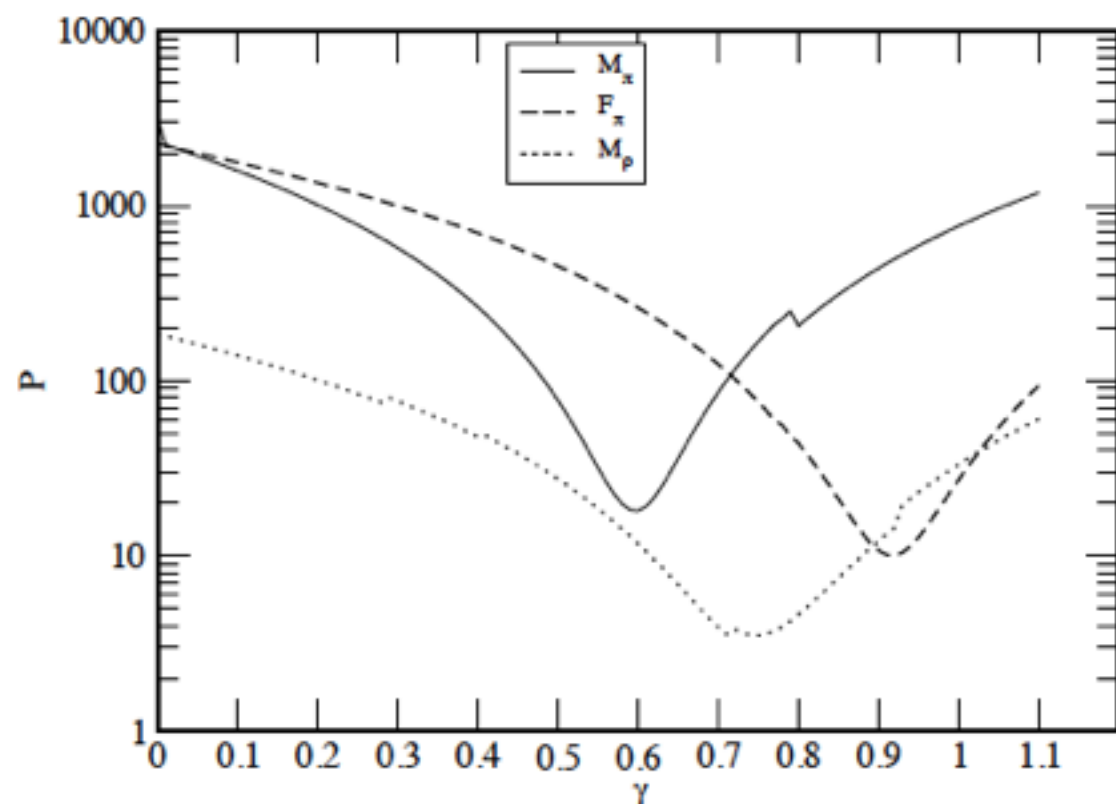
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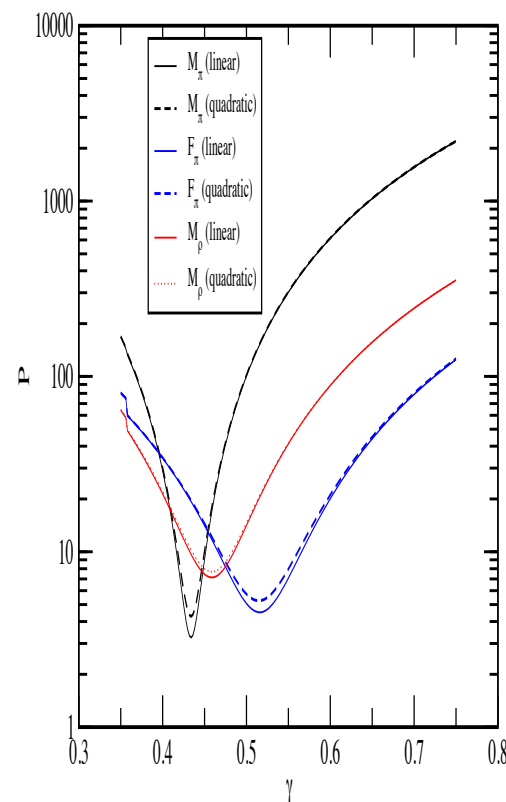
$N_f=8$

quantity	γ
M_π	0.593(2)
F_π	0.955(4)
M_ρ	0.820(20)



- Optimal γ obtained for each quantity

$P(\gamma)$ analysis for $N_f=8$



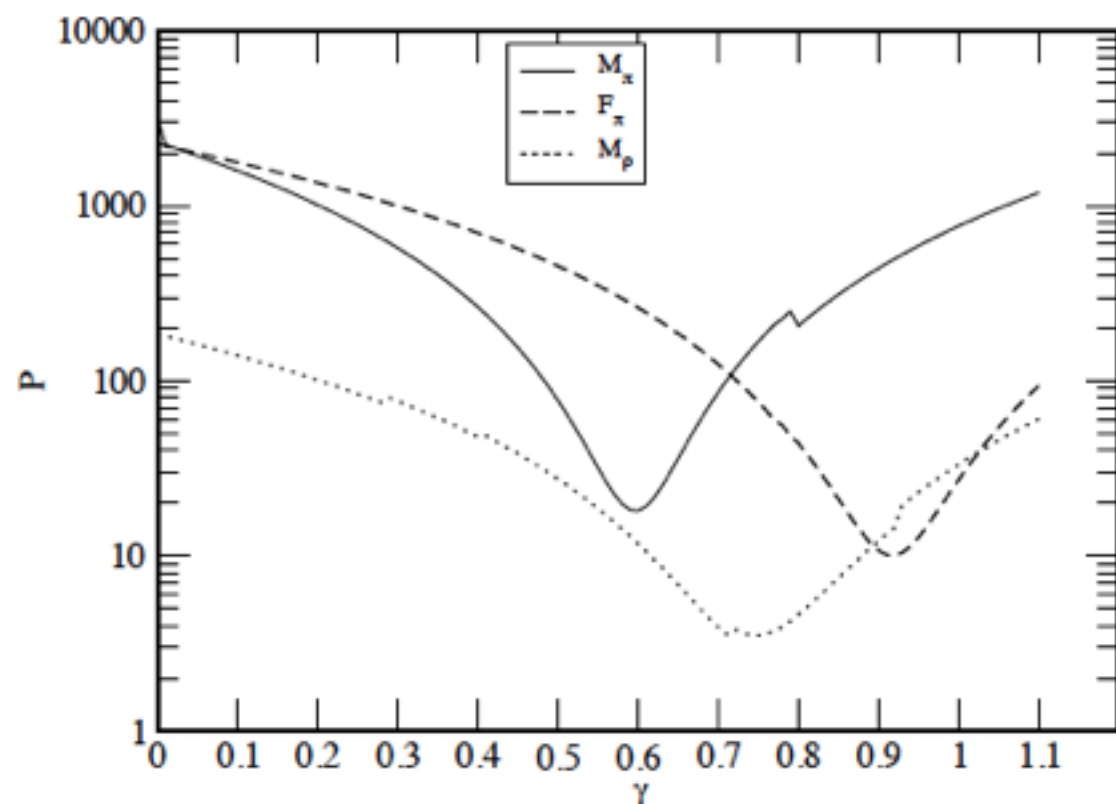
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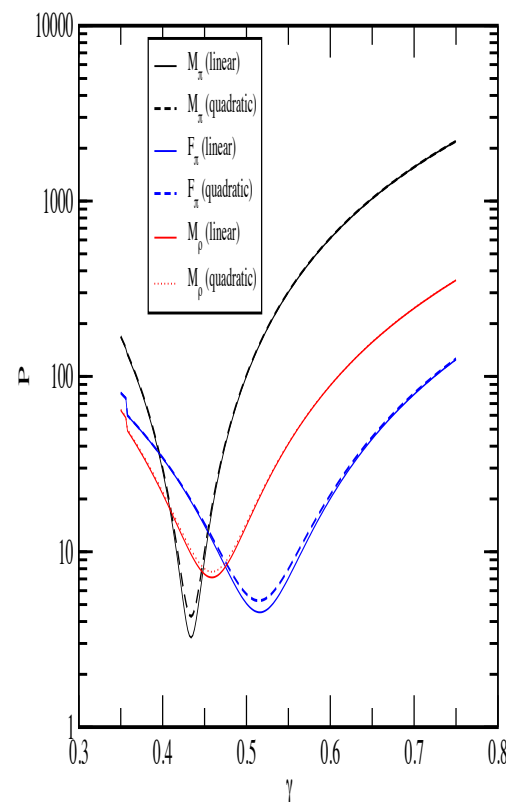
$N_f=8$

quantity	γ
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M_ρ	0.820(20)



- Optimal γ obtained for each quantity
- γ scattered \rightarrow no exact conformality

$P(\gamma)$ analysis for $N_f=8$



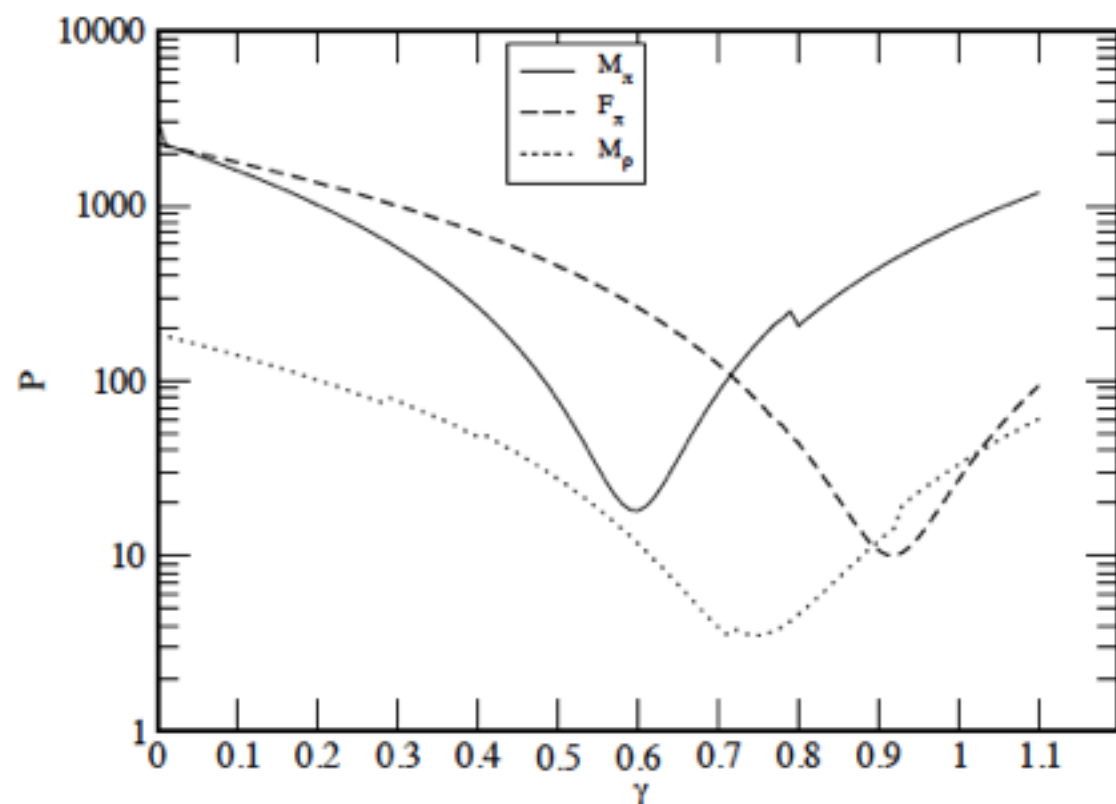
$N_f=12$

quantity	γ
M_π	0.434(4)
F_π	0.516(12)
M_ρ	0.459(8)

statistical error only

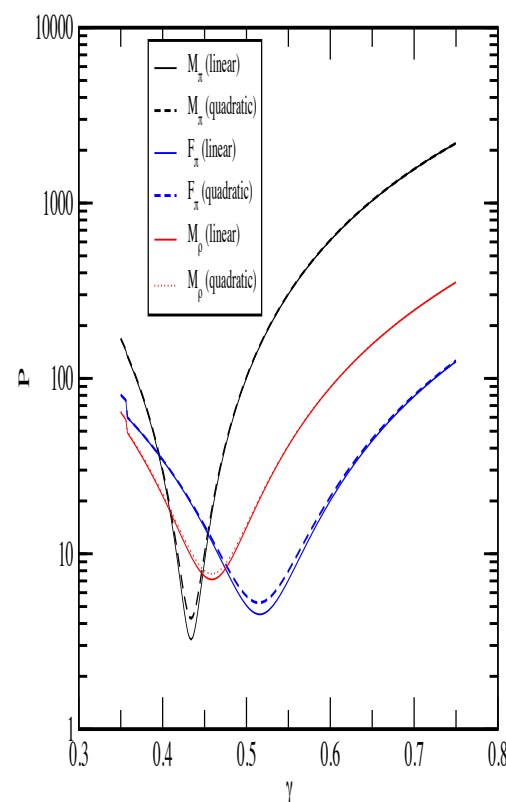
$N_f=8$

quantity	γ
M_π	0.593(2)
F_π	0.955(4)
M_ρ	0.820(20)



- Optimal γ obtained for each quantity
- γ scattered \rightarrow no exact conformality
- scaling \rightarrow remnant conformality

$P(\gamma)$ analysis for $N_f=8$



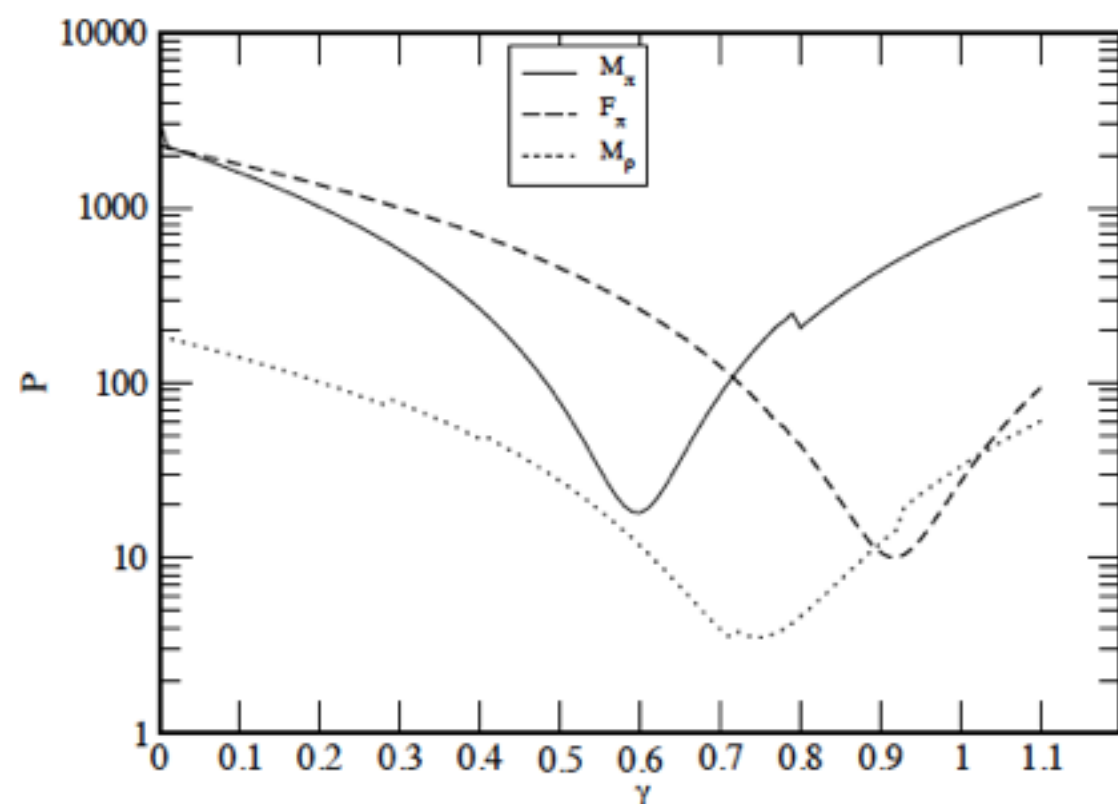
$N_f=12$

quantity	γ
M_π	0.434(4)
F_π	0.516(12)
M_ρ	0.459(8)

statistical error only

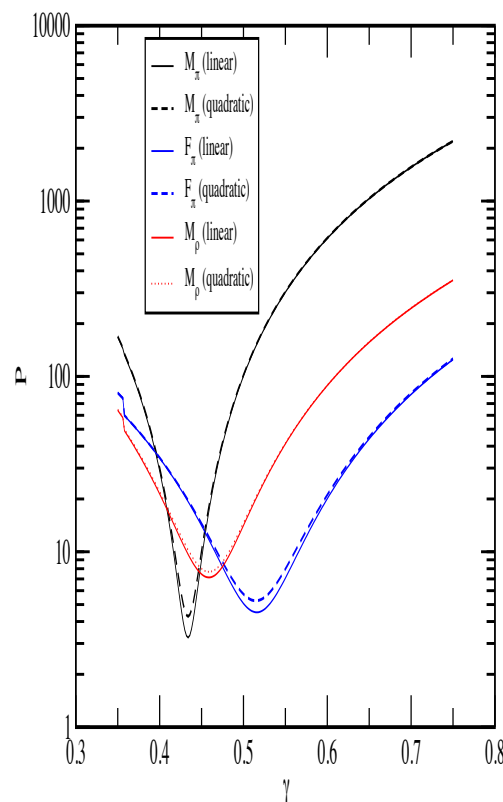
$N_f=8$

quantity	γ
M_π	0.593(2)
F_π	0.955(4)
M_ρ	0.820(20)



- Optimal γ obtained for each quantity
- γ scattered \rightarrow no exact conformality
- scaling \rightarrow remnant conformality
- remember: ~~chiral symmetry~~

$P(\gamma)$ analysis for $N_f=8$



$N_f=12$

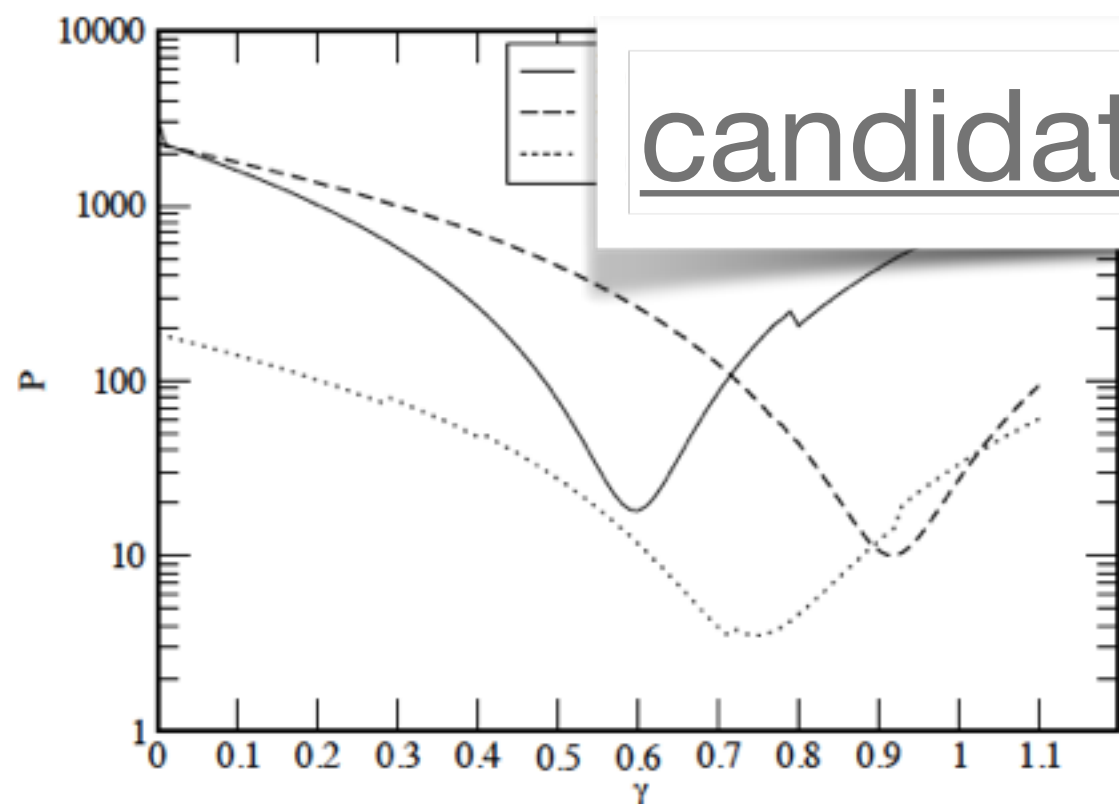
quantity	γ
M_π	0.434(4)
F_π	0.516(12)
M_ρ	0.459(8)

statistical error only

$N_f=8$

quantity	γ
M_π	0.593(2)
F_π	0.5(4)
M_ρ	0.9(20)

candidate of walking TC



- Optimal γ obtained for each quantity
- γ scattered \rightarrow no exact conformality
- scaling \rightarrow remnant conformality
- remember: ~~chiral symmetry~~

TABLE VII. Summary of the optimal values of γ . See the text for details.

quantity	β	all
M_π	3.7	0.434(4)
F_π	3.7	0.516(12)
M_ρ	3.7	0.459(8)

TABLE VII. Summary of the optimal values of γ . See the text for details.

quantity	β	all	x		
			range 1	range 2	range 3
M_π	3.7	0.434(4)	0.425(9)	0.436(6)	0.437(4)
F_π	3.7	0.516(12)	0.481(19)	0.512(19)	0.544(14)
M_ρ	3.7	0.459(8)	0.411(17)	0.461(10)	0.473(8)

TABLE VII. Summary of the optimal values of γ . See the text for details.

quantity	β	all	x		
			range 1	range 2	range 3
M_π	3.7	0.434(4)	0.425(9)	0.436(6)	0.437(4)
F_π	3.7	0.516(12)	0.481(19)	0.512(19)	0.544(14)
M_ρ	3.7	0.459(8)	0.411(17)	0.461(10)	0.473(8)

• $\beta=3.7$: smaller m : closer to M_π

TABLE VII. Summary of the optimal values of γ . See the text for details.

quantity	β	all	x			L		
			range 1	range 2	range 3	(18,24)	(18,30)	(24,30)
M_π	3.7	0.434(4)	0.425(9)	0.436(6)	0.437(4)	0.438(6)	0.433(4)	0.429(8)
F_π	3.7	0.516(12)	0.481(19)	0.512(19)	0.544(14)	0.526(18)	0.514(11)	0.505(24)
M_ρ	3.7	0.459(8)	0.411(17)	0.461(10)	0.473(8)	0.491(15)	0.457(8)	0.414(18)

• $\beta=3.7$: smaller m : closer to M_π

TABLE VII. Summary of the optimal values of γ . See the text for details.

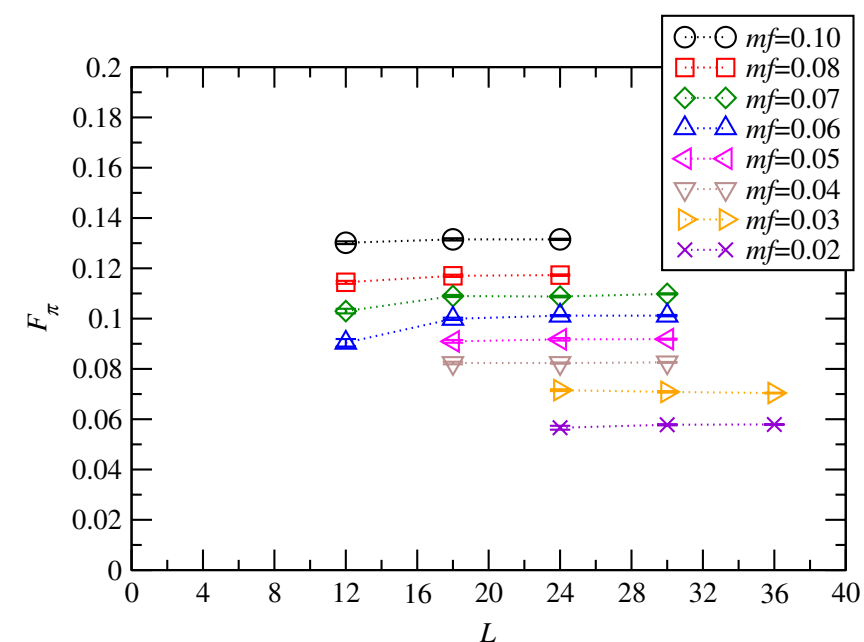
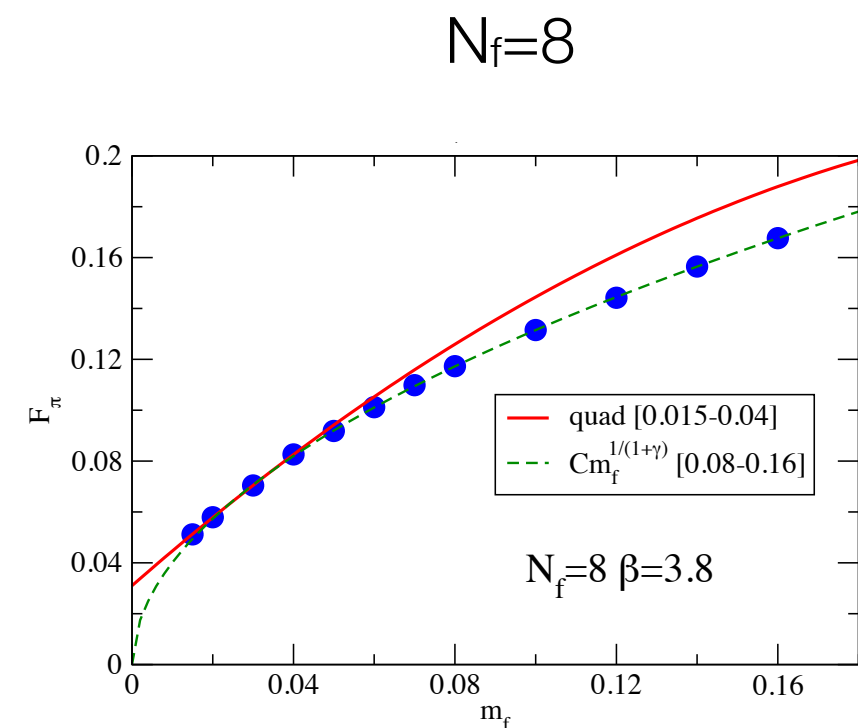
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- $\beta=3.7$: smaller m : closer to M_π
- $\beta=3.7$: larger V : closer to M_π
-

Nf=8: including smaller mf → scaling gets worse

$$F_\pi = C_1 m_f^{1/(1+\gamma)}$$

Fit range (m_f)	C_1	γ	χ^2/dof
0.015–0.04	0.415(7)	0.988(19)	14.8
0.015–0.05	0.414(5)	0.991(15)	9.84
0.015–0.06	0.418(4)	0.979(12)	7.88
0.015–0.07	0.424(3)	0.963(9)	7.35
0.015–0.08	0.425(3)	0.961(8)	6.15
0.015–0.10	0.426(2)	0.958(7)	5.31
0.015–0.16	0.428(1)	0.952(4)	3.98
0.02–0.16	0.429(1)	0.947(4)	2.22
0.03–0.16	0.431(1)	0.942(5)	1.94
0.04–0.16	0.429(2)	0.950(10)	1.23
0.05–0.16	0.431(2)	0.941(7)	0.66
0.06–0.16	0.429(2)	0.948(9)	0.44
0.07–0.16	0.429(3)	0.950(10)	0.52
0.08–0.16	0.431(3)	0.939(14)	0.20
0.10–0.16	0.432(4)	0.934(19)	0.23



$N_f=8$: an interpretation

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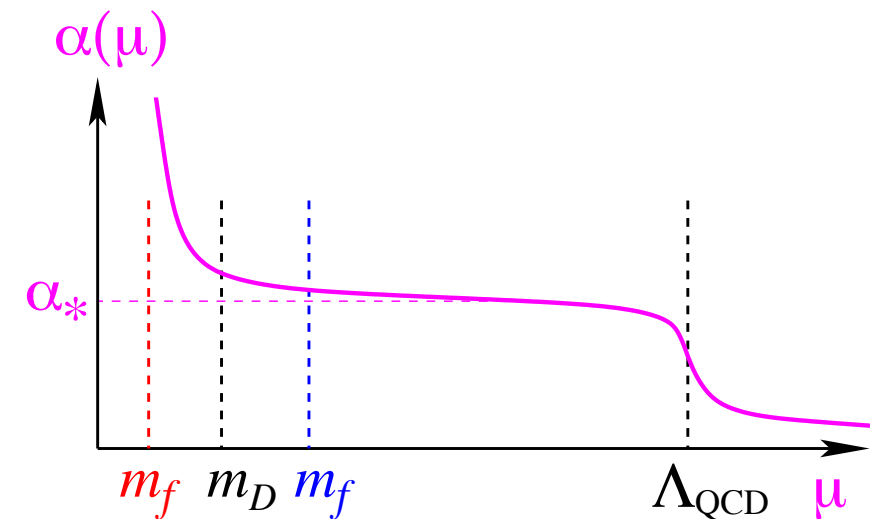
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 - hyperscaling for intermediate m_f
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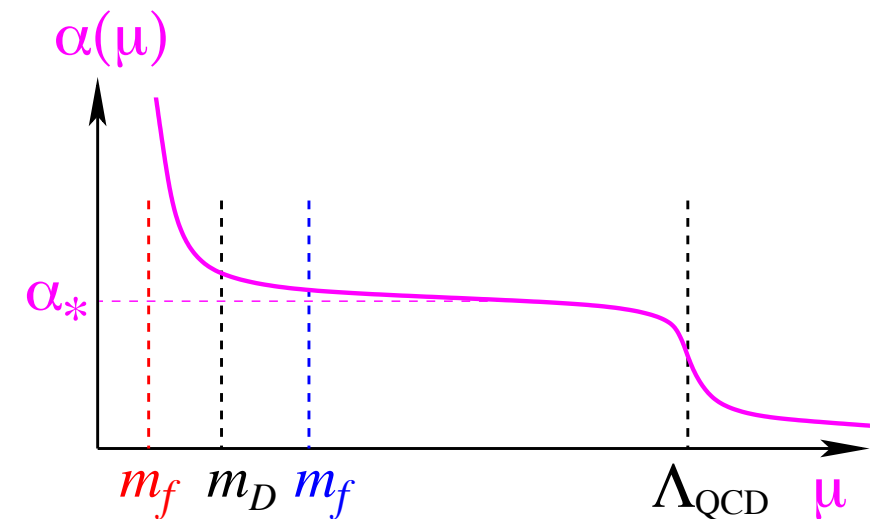
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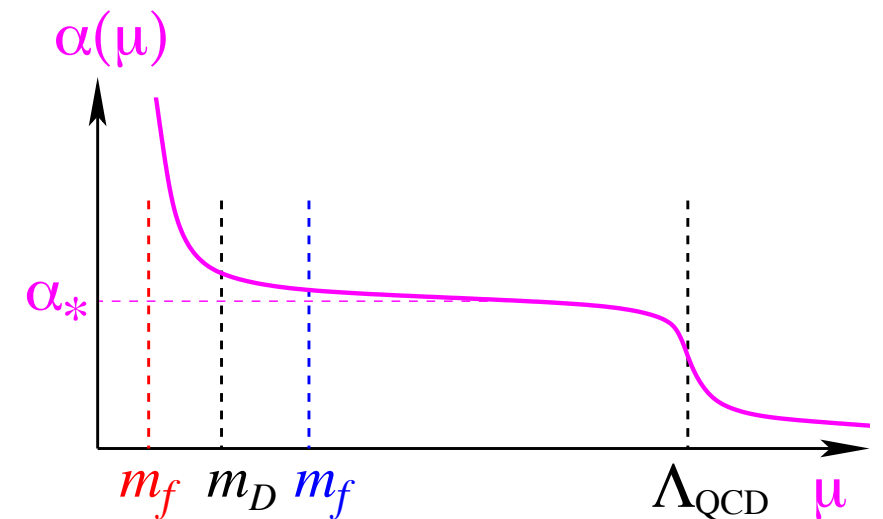
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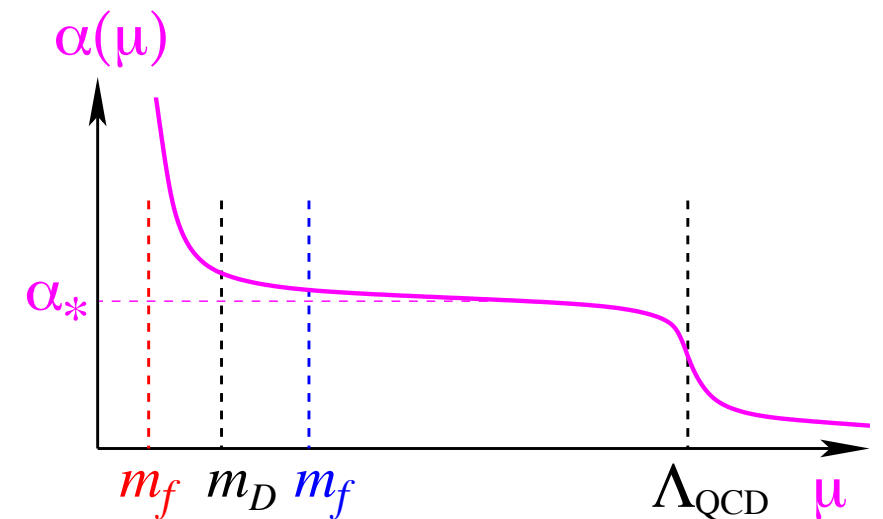
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- walking: a solution to classical technicolor problem: quark mass \leftrightarrow FCNC
- Next interesting direction \rightarrow prediction (postdiction) of spectrum



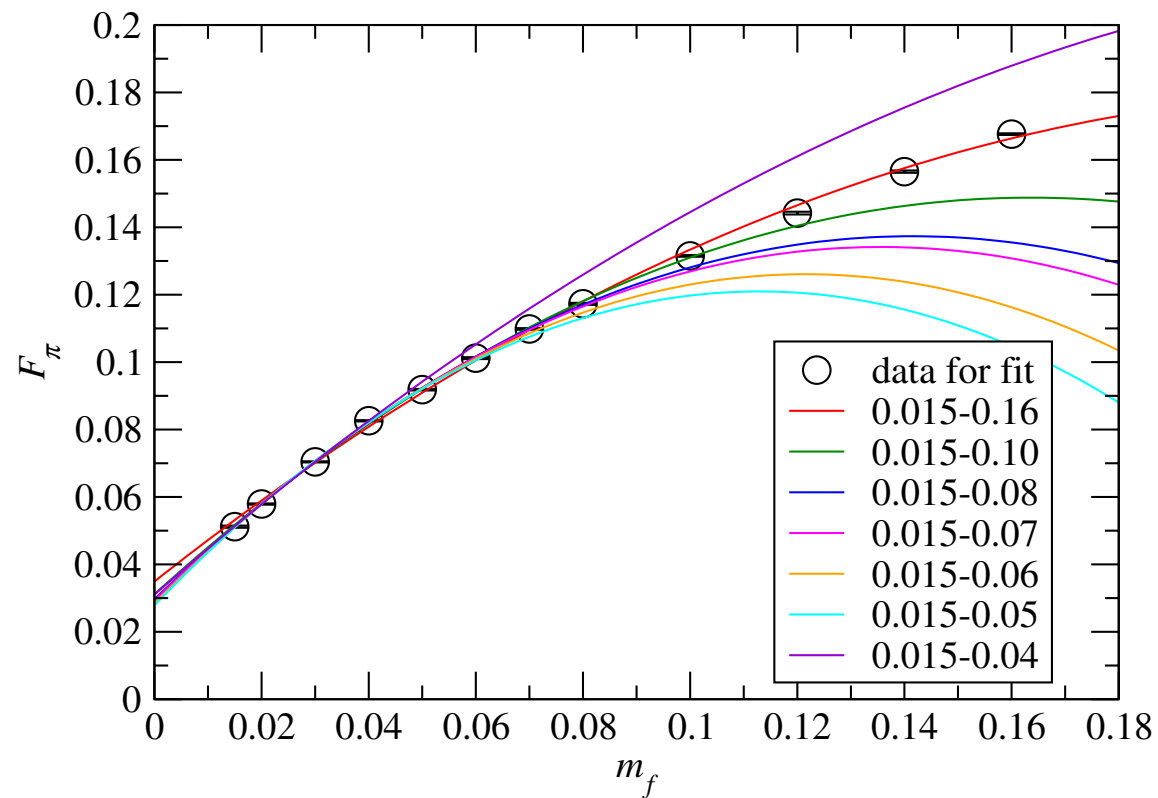


FIG. 8 (color online). Results of quadratic fit of F_π for various fit ranges.

$$F = 0.031(1) \begin{pmatrix} +2 \\ -10 \end{pmatrix},$$

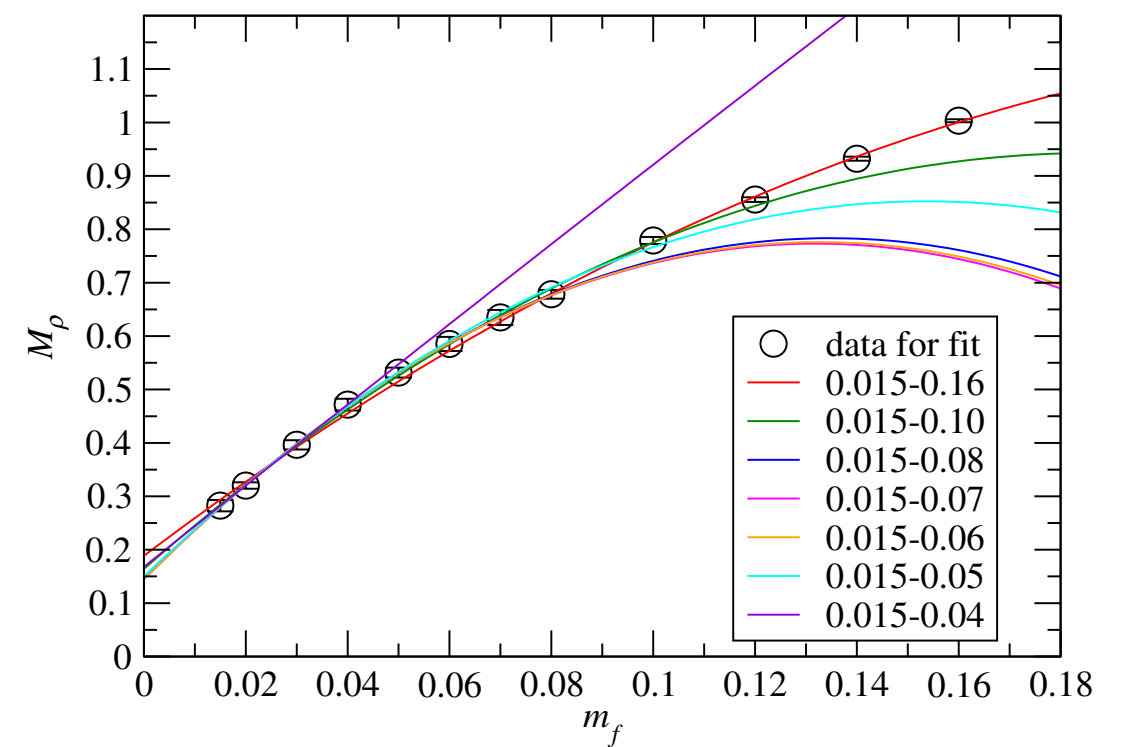


FIG. 9 (color online). Results of quadratic fit of M_ρ for various fit ranges.

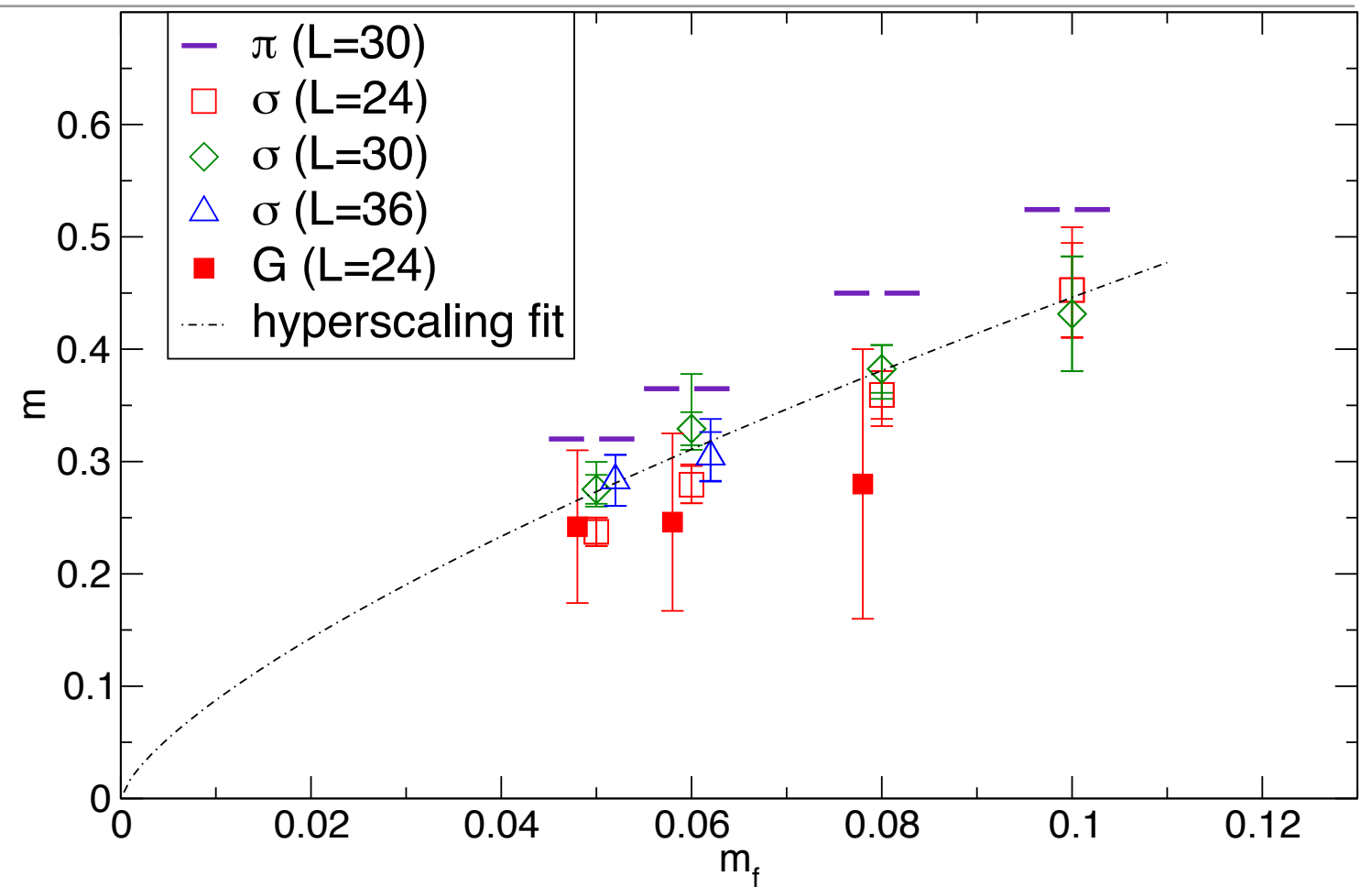
$$M_\rho = 0.168(32).$$

- chiral log correction included in the systematic error of F

$N_f=8$ spectrum

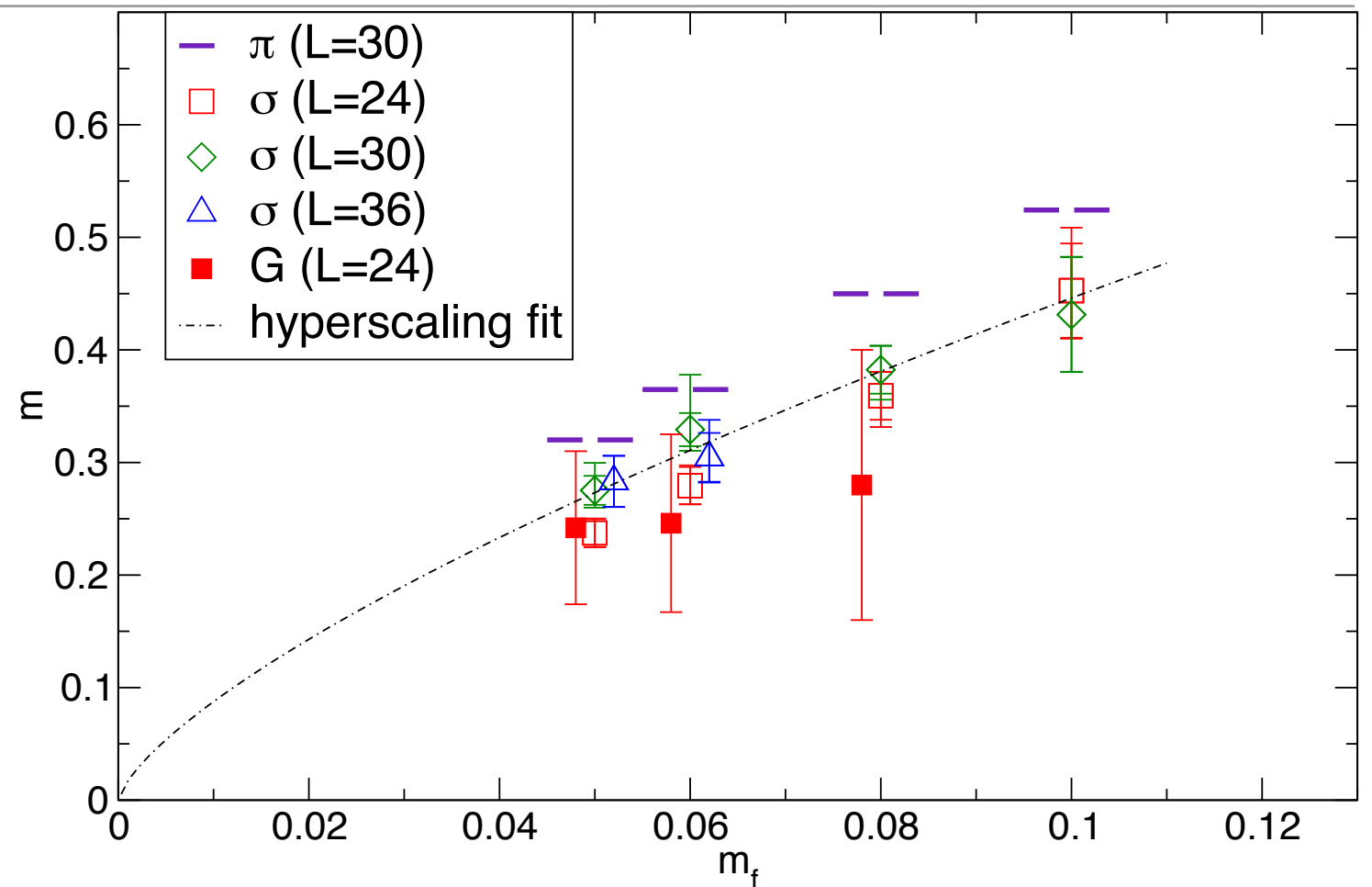
- with input $F_\pi = 246 / \sqrt{N}$ GeV (N: # weak doublet in techni-sector)
 - prediction: $M_\rho / F_\pi = 7.7(1.5)(^{+3.8}_{-0.4})$ (with only technicolor dynamics)
 - for example: $M_\rho = 970(^{+515}_{-195})$ GeV for one family model: N=4
 - Higgs mass ?
 - 125 GeV (LHC) seems very light for technicolor
 - 0^{++} : one of the difficult quantities on the lattice
 - multi-faceted nature of $N_f=8$ adds another difficulty: delicate chiral extrapl.
- ➡ first analyze simpler $N_f=12$, which shares “conformality” → techni dilaton
- ➡ Is 0^{++} state light in (mass deformed) $N_f=12$ theory ?

flavor singlet scalar spectrum in $N_f=12$ theory



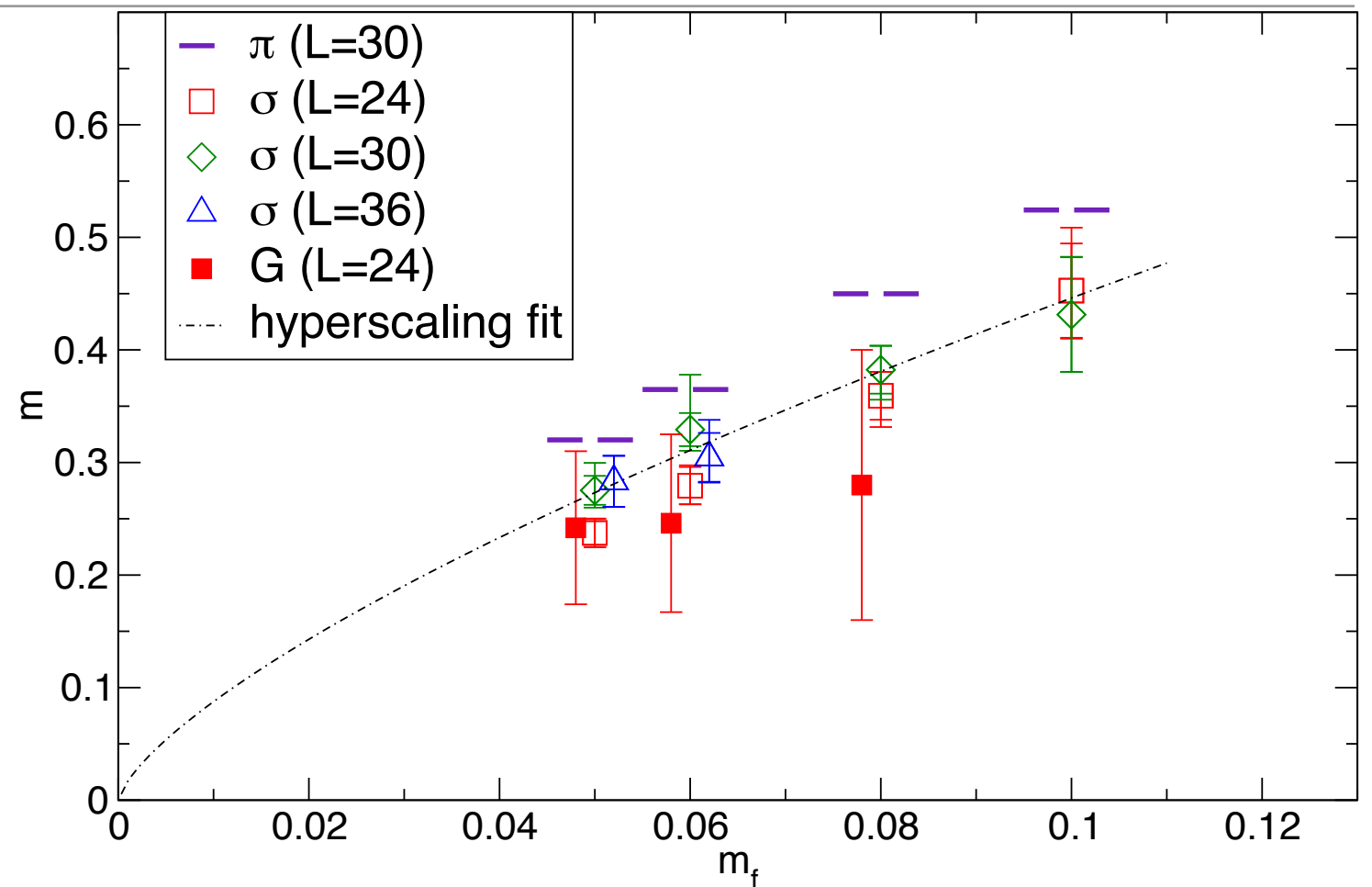
flavor singlet scalar spectrum in $N_f=12$ theory

- with very high statistics
- and a variance reduction
- we got a reasonable signal



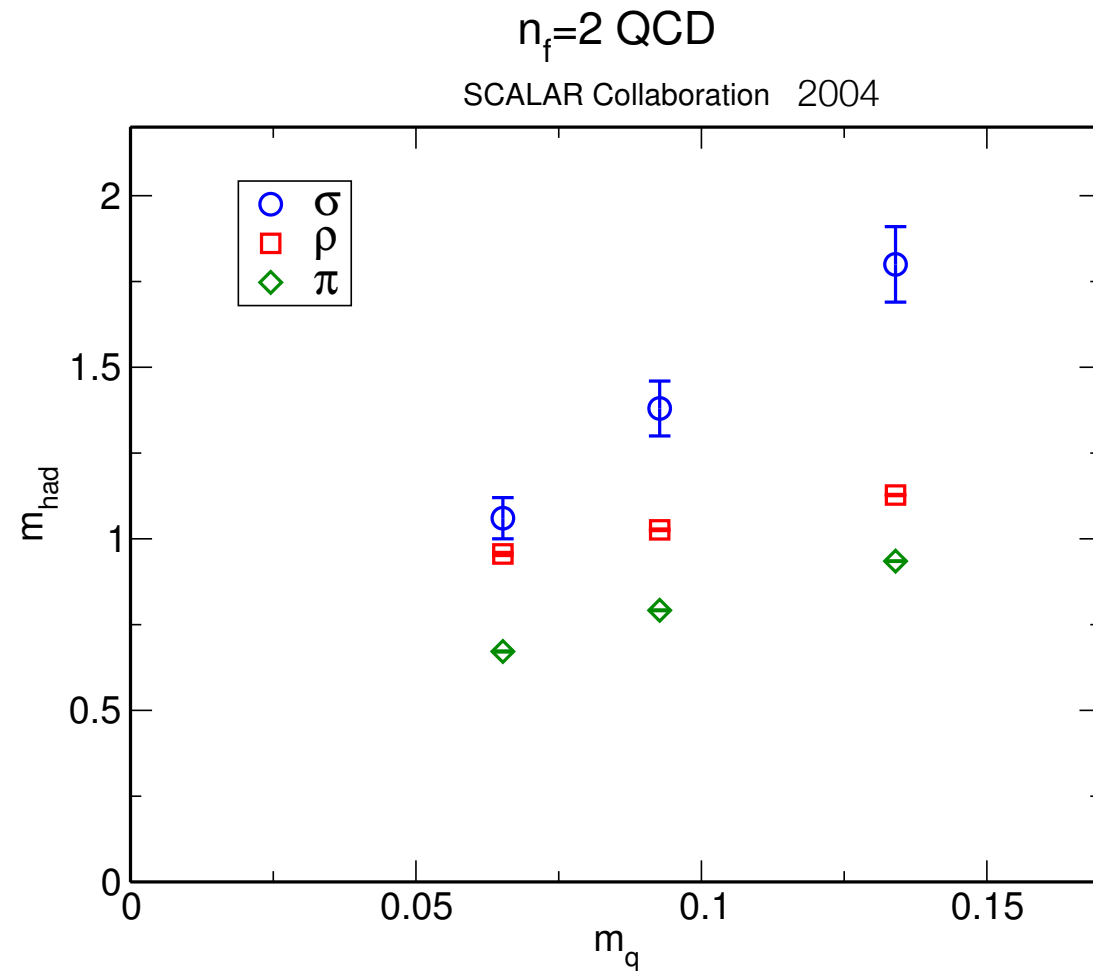
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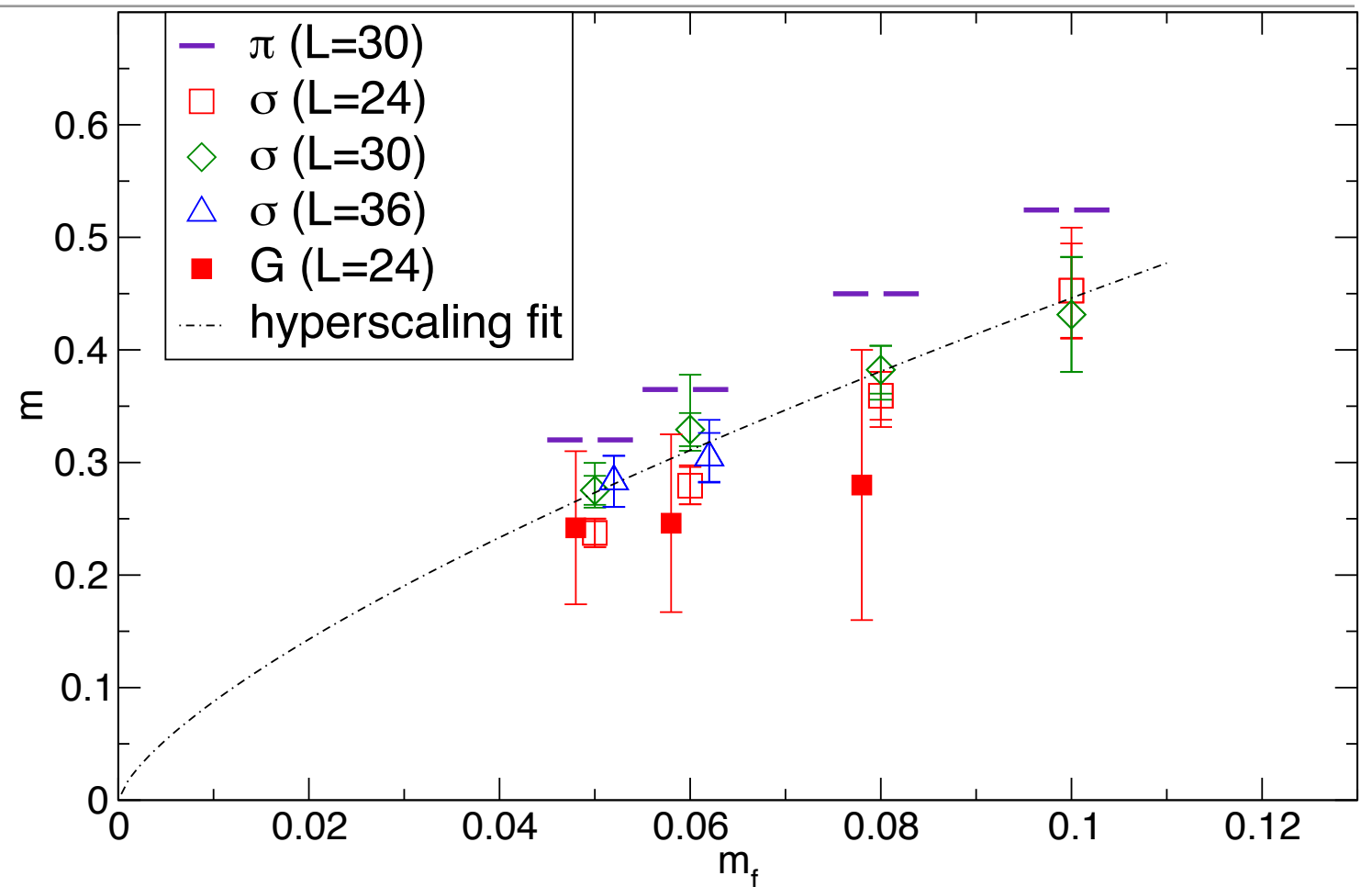
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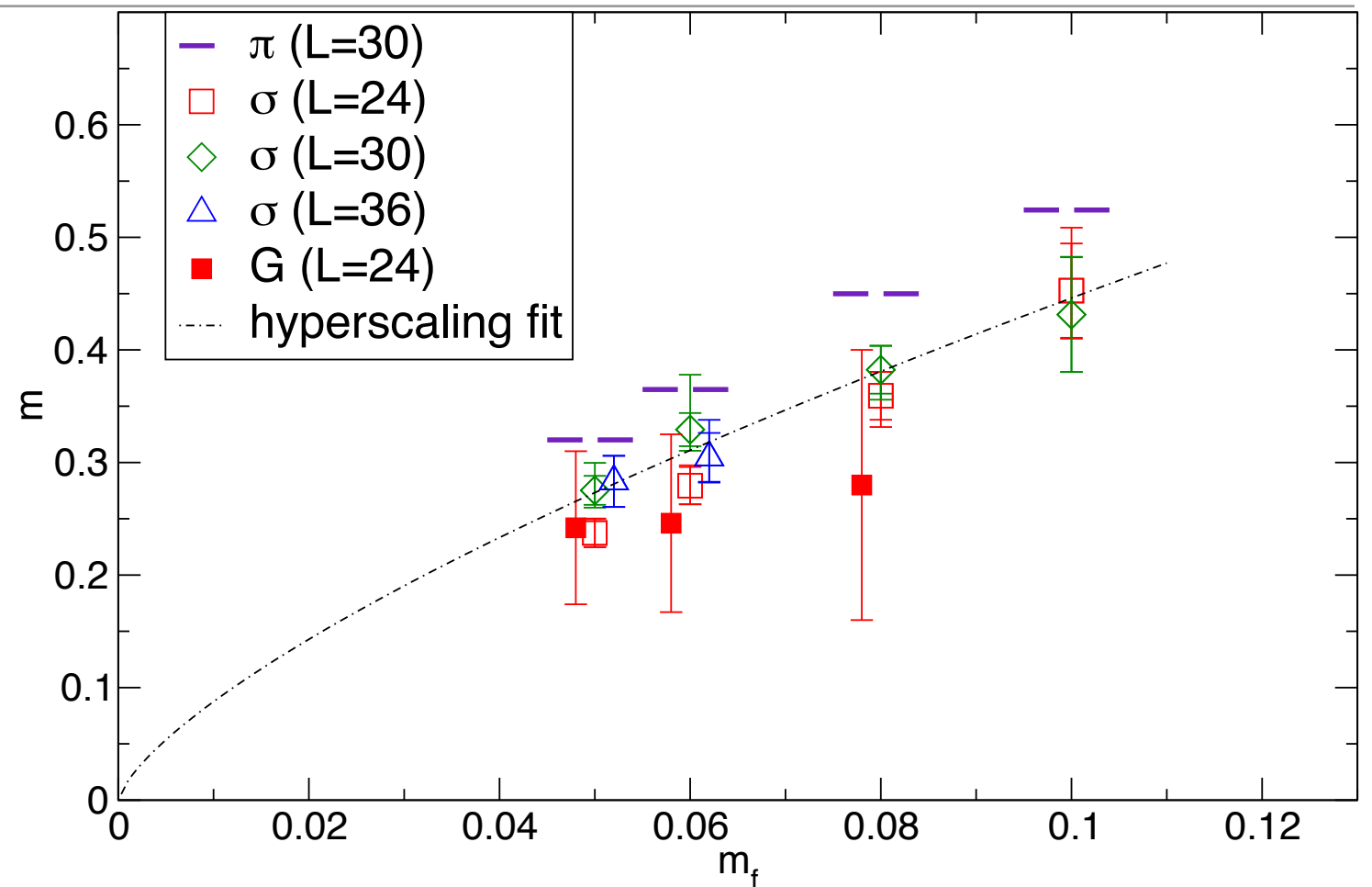
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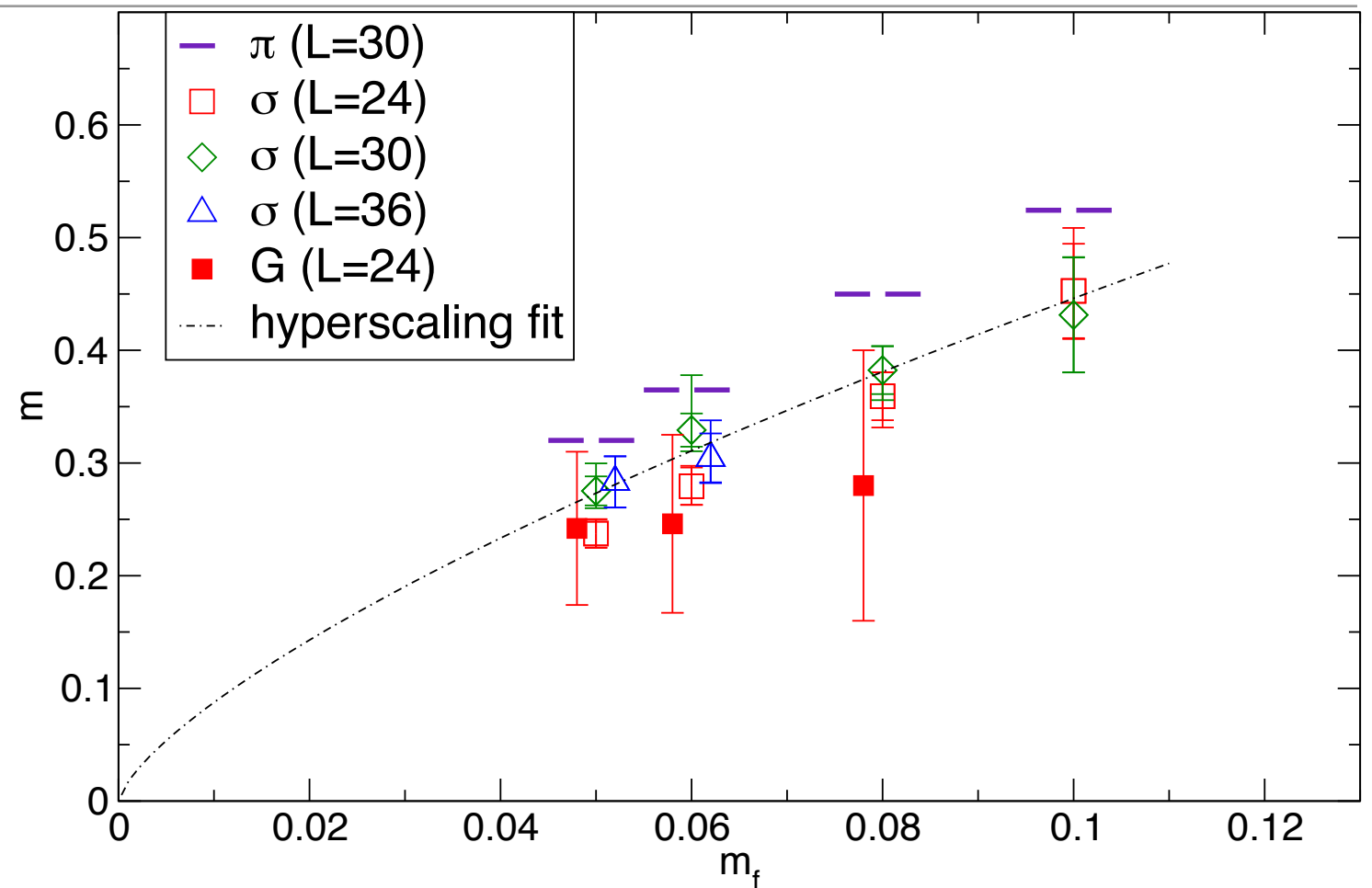
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 - 0^{++} glueball is lightest for SU(2) $n_f=2$ adjoint [Del Debbio et al, 2010]



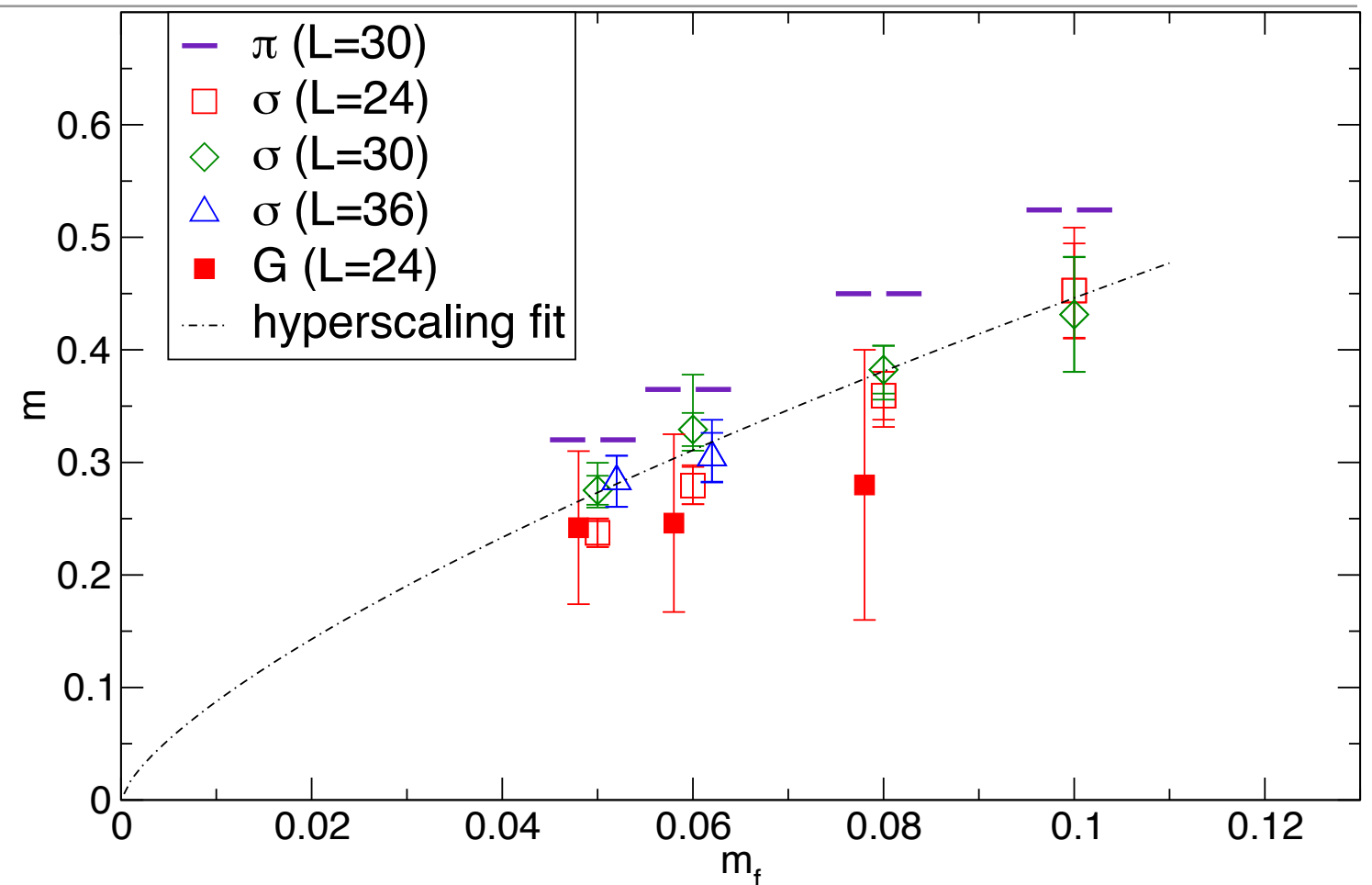
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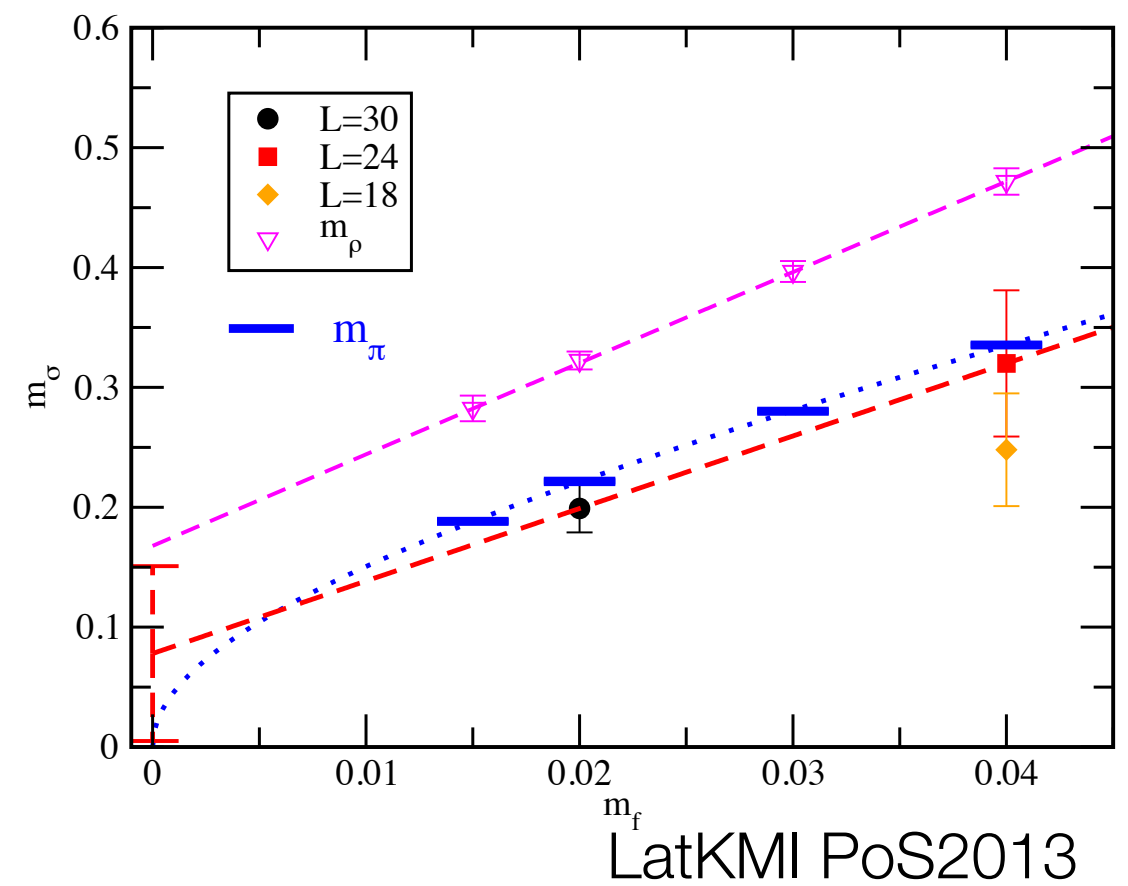


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- LatKMI, PRL 111 (2013), “Light composite scalar in twelve-flavor QCD on the lattice”

scalar (Higgs) in $N_f=8$ theory

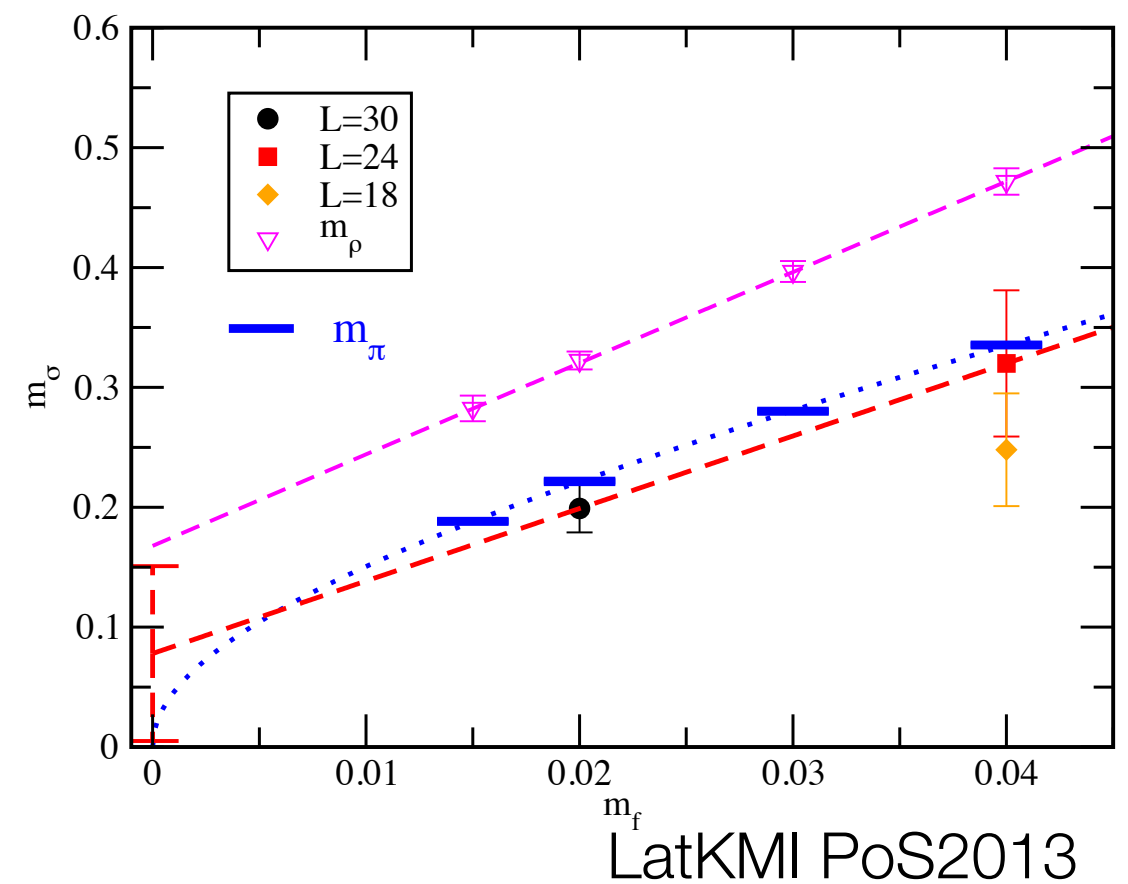
scalar (Higgs) in $N_f=8$ theory

- preliminary results reported at Lattice 2013



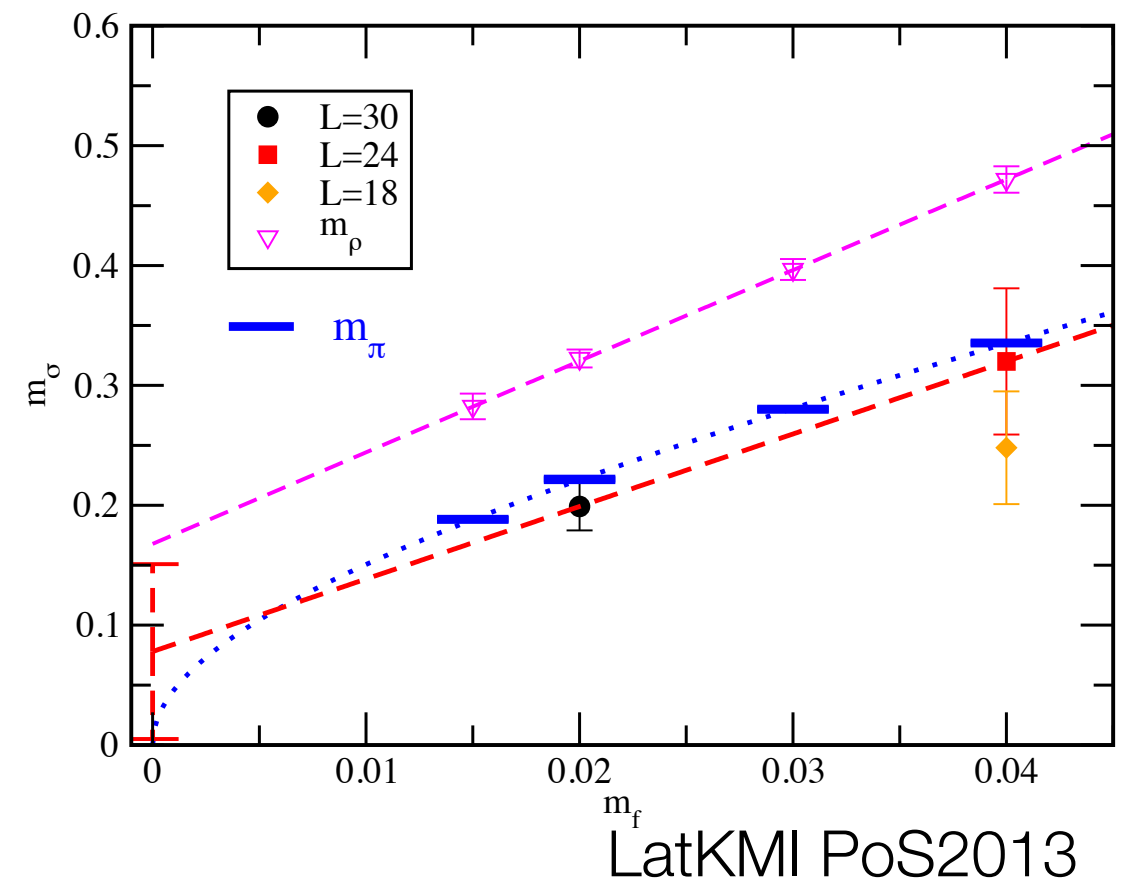
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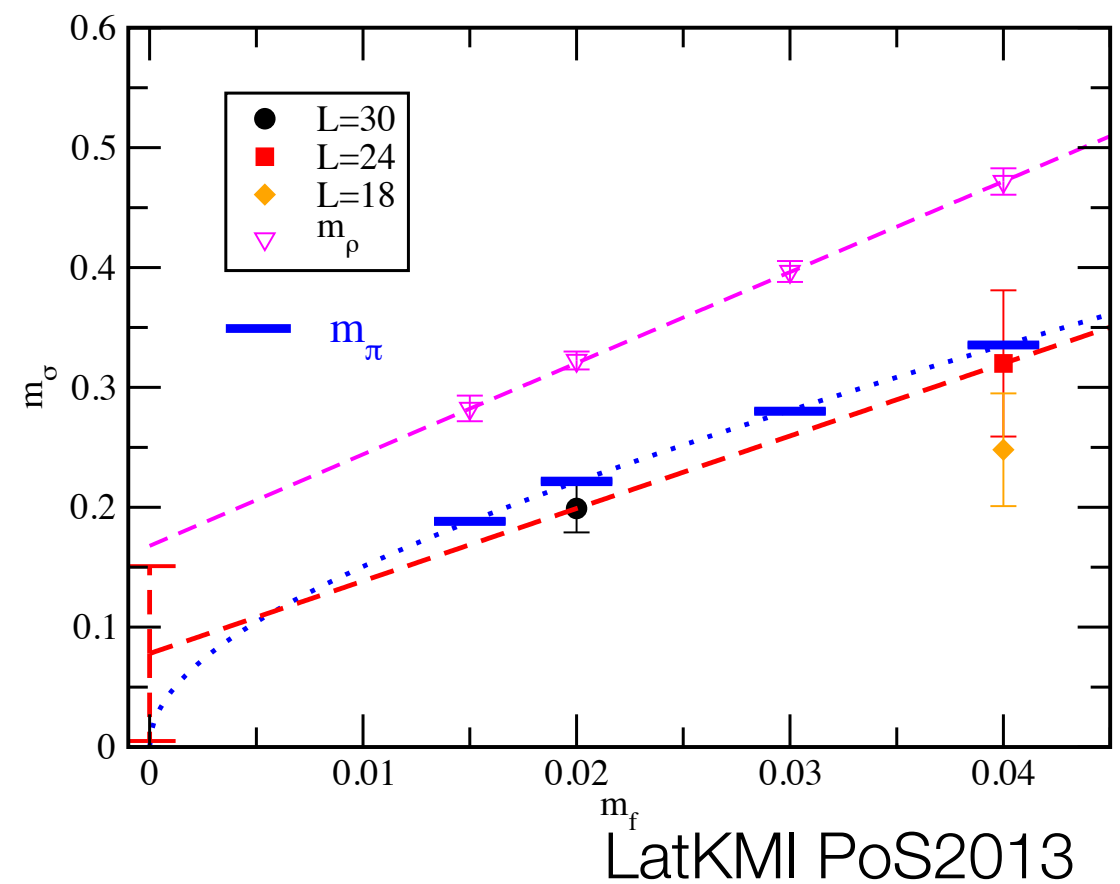
scalar (Higgs) in $N_f=8$ theory

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 - far lighter than ρ



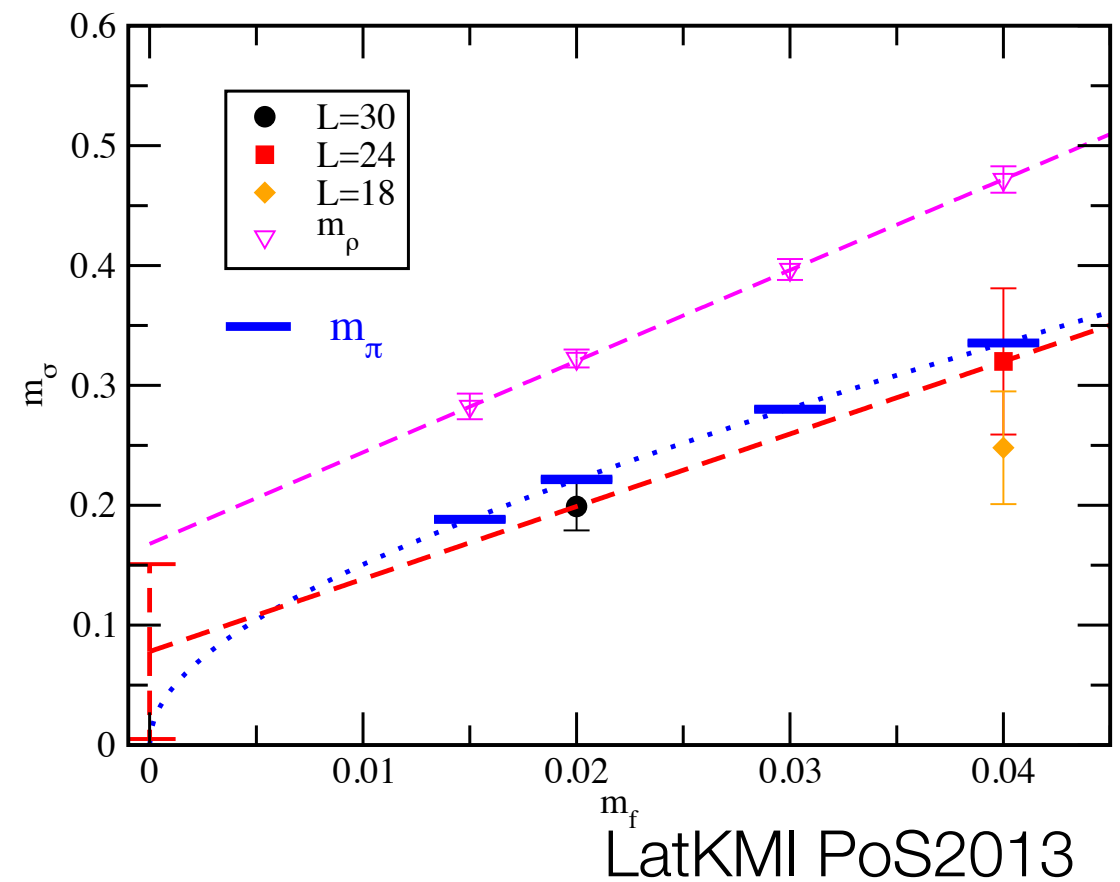
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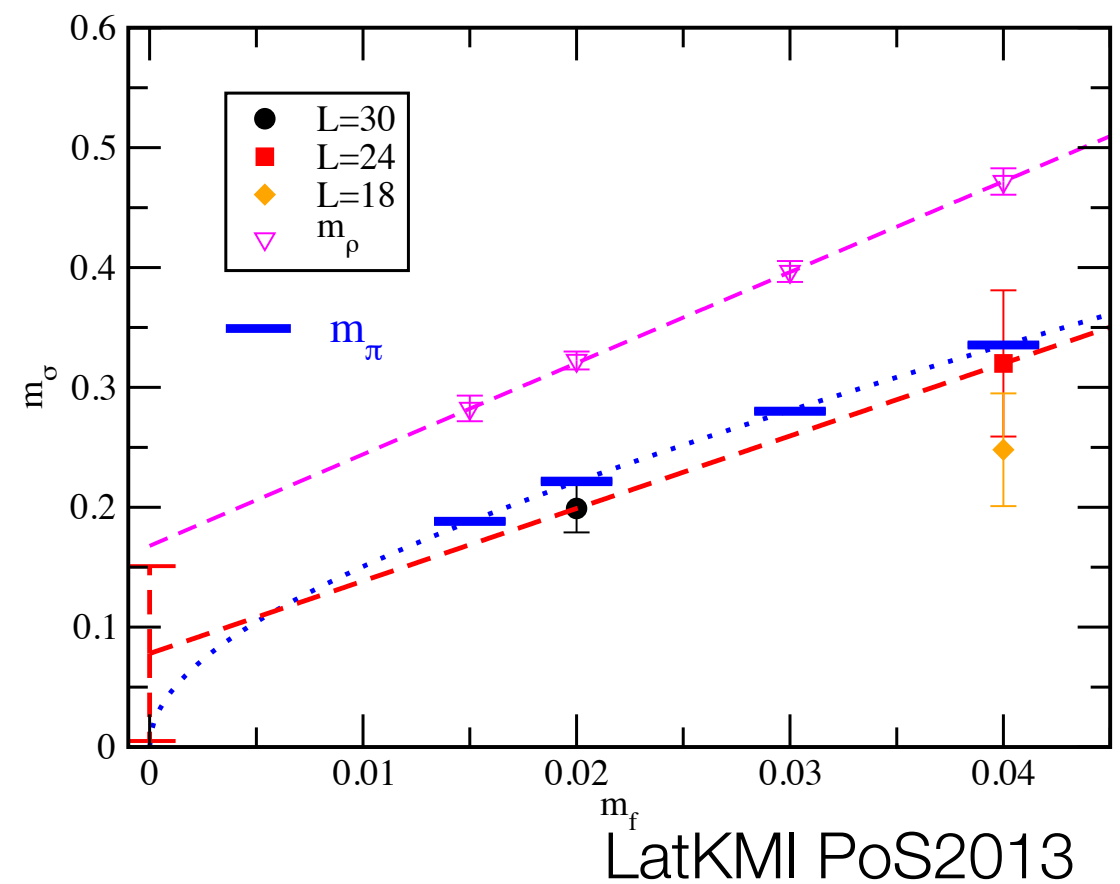
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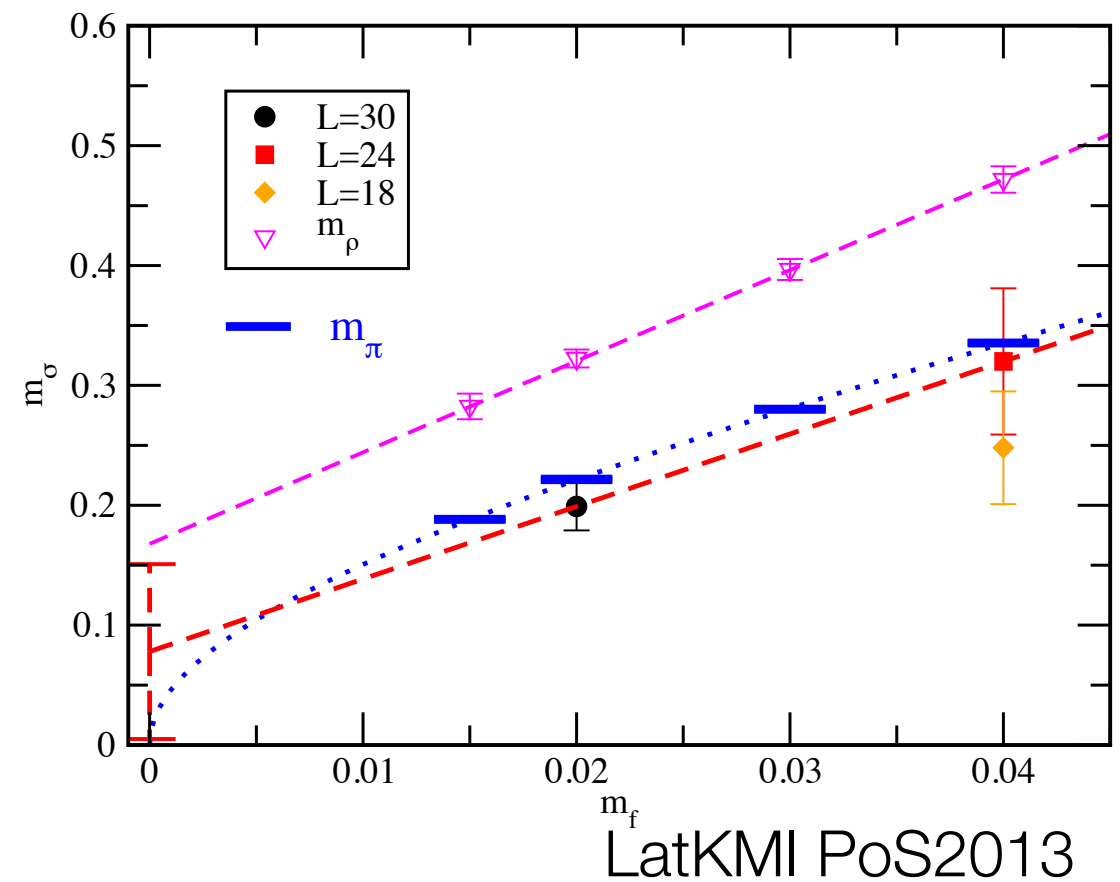
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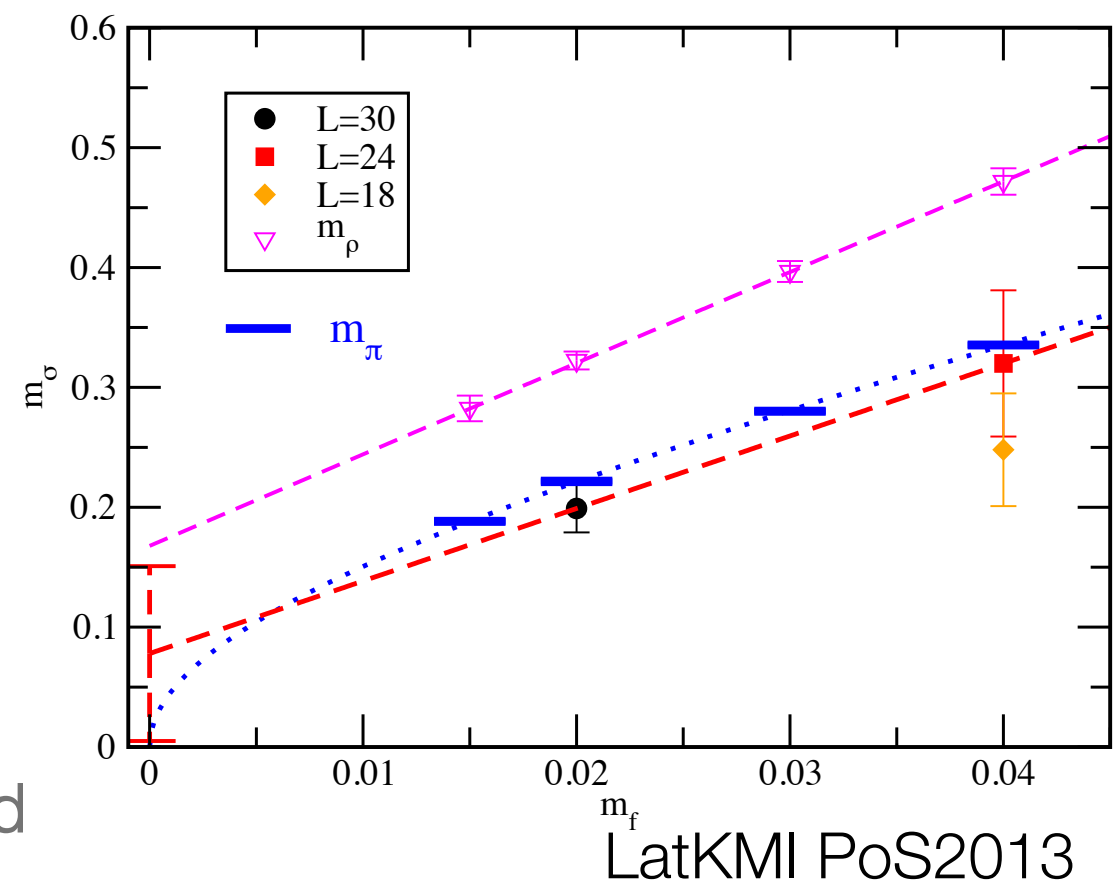
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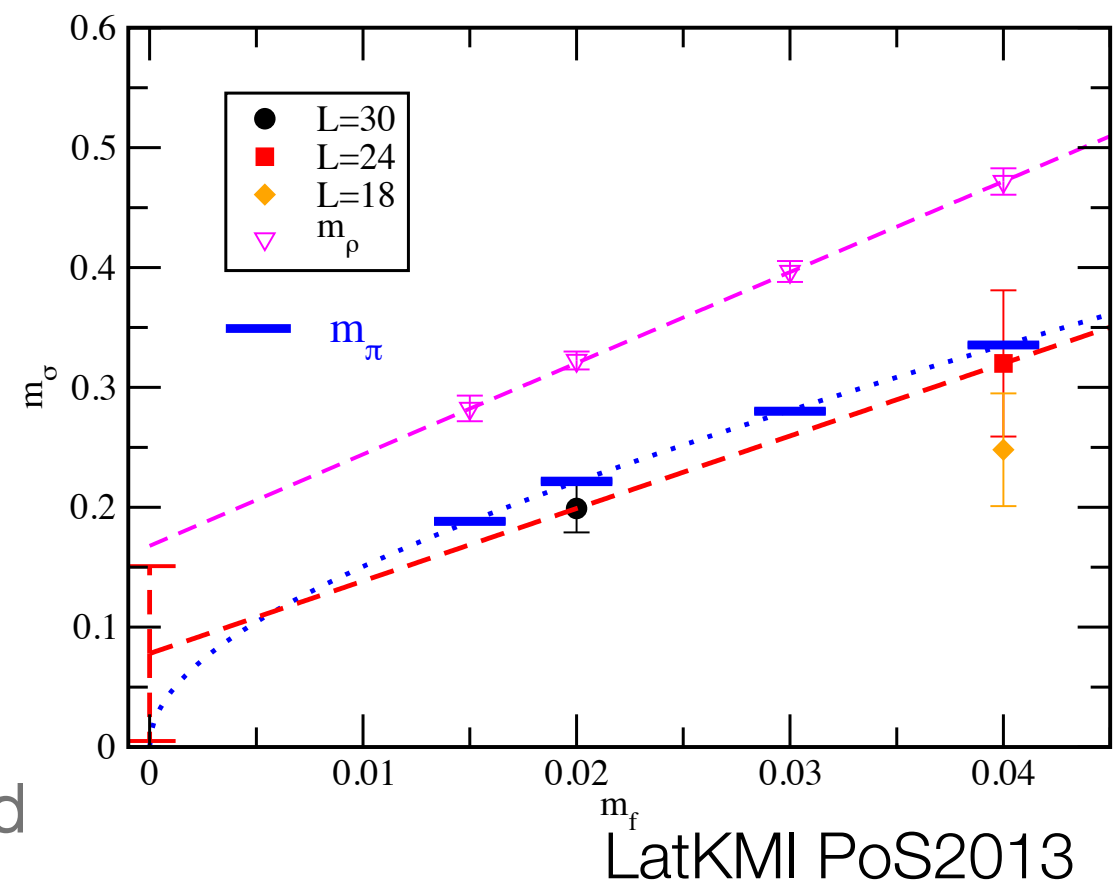
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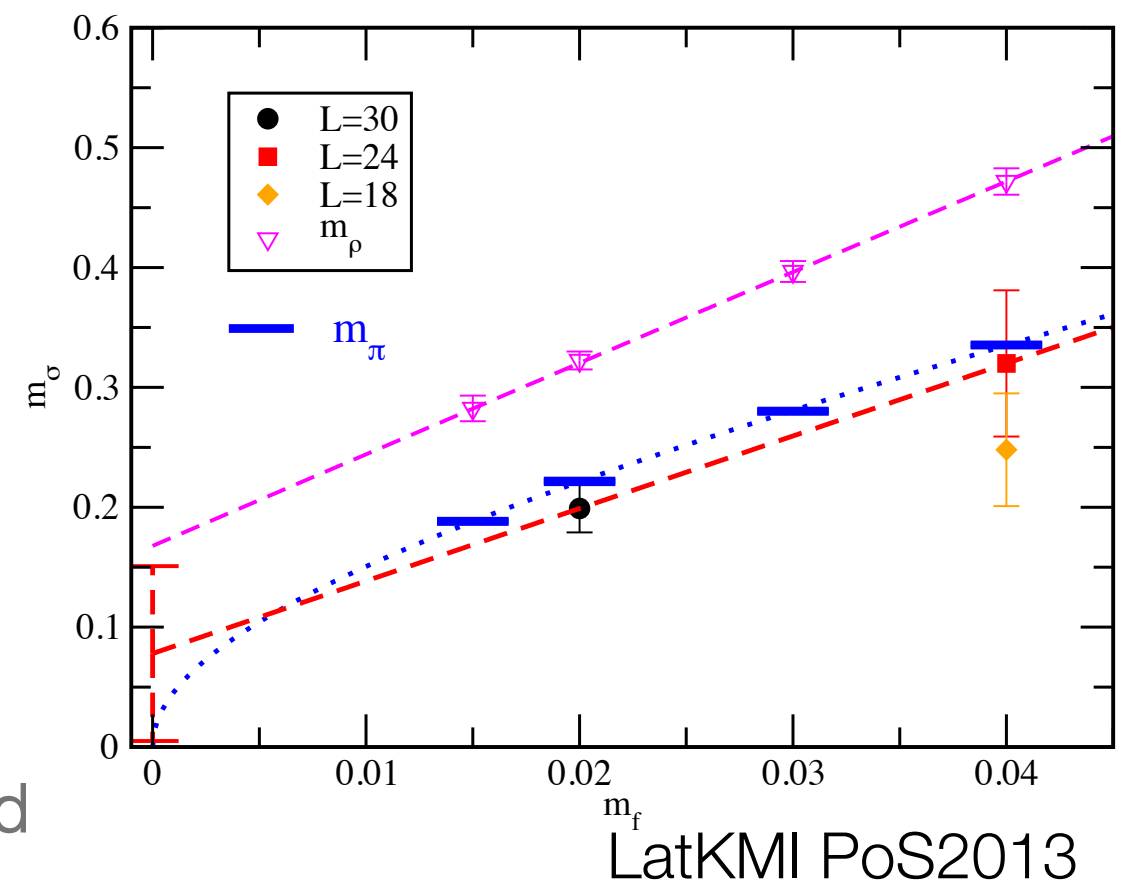
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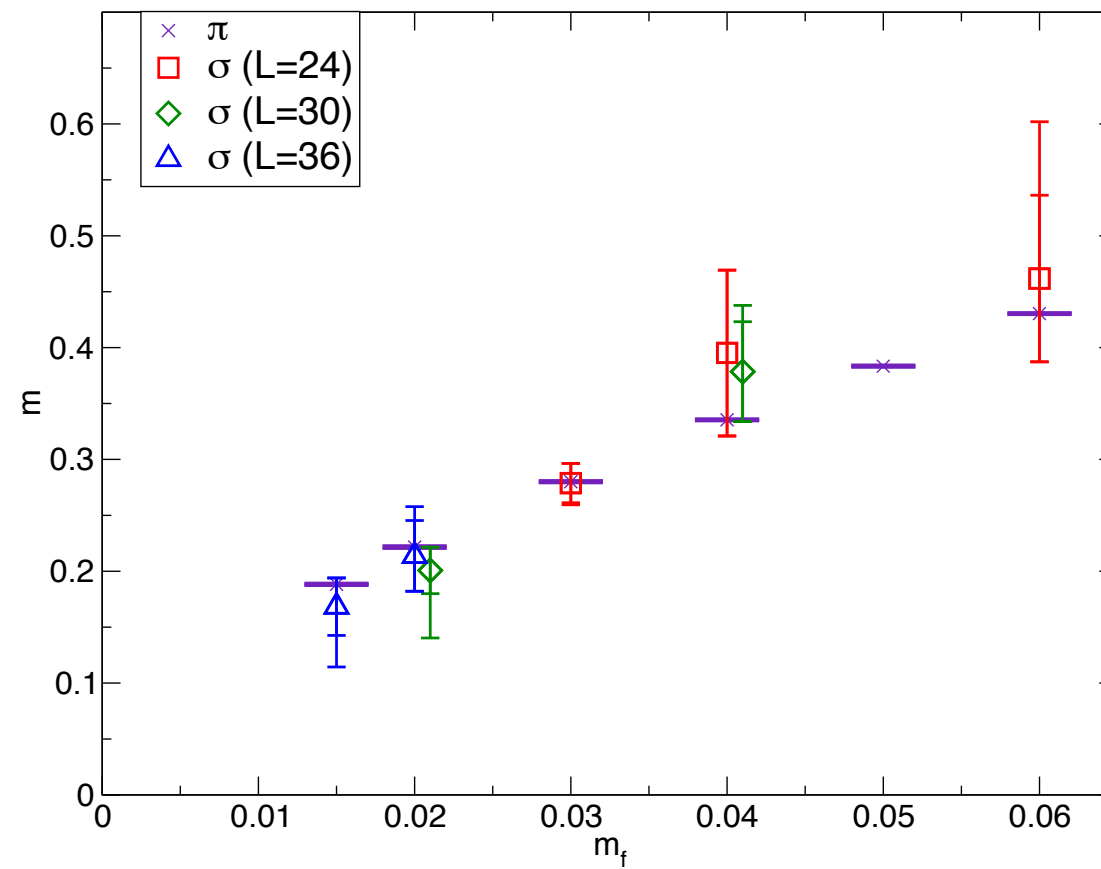


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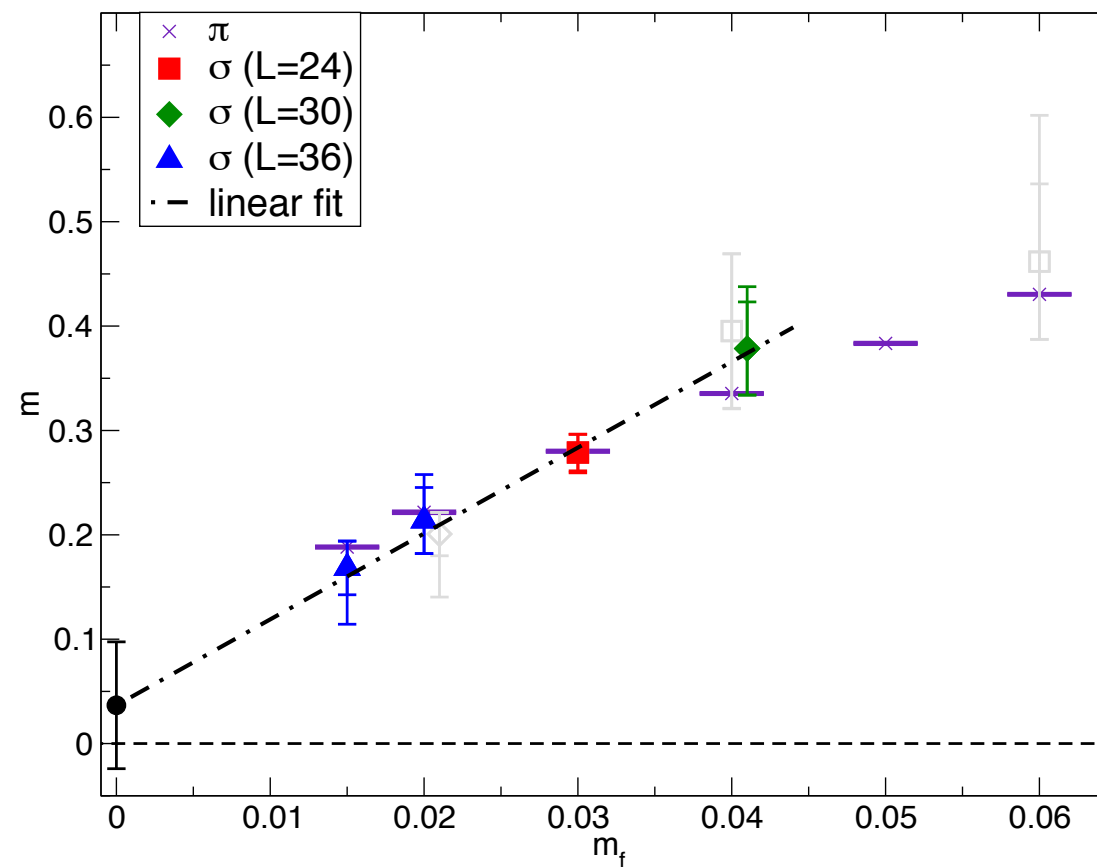
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 - further effort will be required
 - needs a change in chiral effective theory



Nf=8 scalar: Update after Lattice 2013



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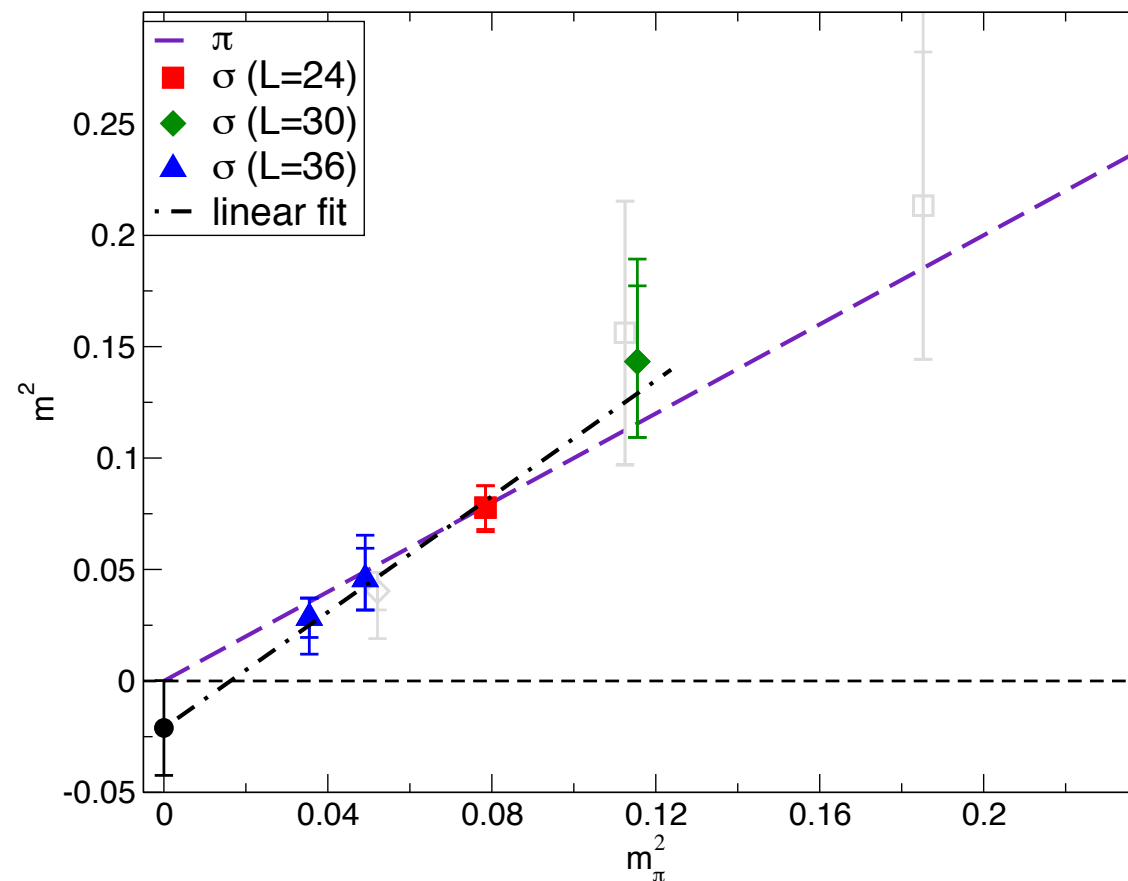
$$m_\sigma = m_0 + Am_f: m_0 = 0.038(61) \rightarrow \frac{m_\sigma}{F} = 1.7(2.8)$$

$$F = 0.0219(7) \text{ PRD87(2013)094511}$$

c.f.) 1 family model: $m_{\text{Higgs}} = 210(340) \text{ GeV}$

Nf=8 scalar: Update after Lattice 2013

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$



ChPT pion and dilaton as
light elements:
Matsuzaki & Yamawaki '13

$m_0^2 < 0$: data not in $m_\sigma > m_\pi$ region

Need data at smaller m_f where $m_\sigma > m_\pi$ as in usual QCD

Summary and Outlook

- LatKMI collaboration is investigating the physics near the conformal phase boundary in $SU(3)$ gauge theory.
- There appears one candidate of walking technicolor theory $N_f=8$ QCD, that could accommodate 125 GeV Higgs found at LHC.
- Solidness of the emerging picture will have to be investigated further:
 - precision needs to be improved
 - controversial pictures (conformality) from different collaborations
- Calculation / technology development for other quantities are underway
 - S parameter: a method proposed for vacuum polarization function
 - low energy parameters in π and σ as effective light elements...

Thank you very much for your attention !